Delhi Public School, Jammu

Question Bank

Session 2017-18

Class $10^{th}$ Subject:-Maths

Q1. Prove that $5-\frac{2}{7}\sqrt{3}$ is not a rational number.

Q2.If HCF of 210 and 55 is expressible in the form of 210 ×5 + 55y, then find y.

Q3.If 1 is zero of polynomial P(x)=$ax^2 - 3(a-1)x - 1$, find a.

Q4.If $\alpha$ and $\beta$ are zeroes of polynomial $3x^2 + 2x - 6$, then find the value of (a) $\alpha^2 + \beta^2$ (b) $\alpha^3 + \beta^3$ (c) $\frac{1}{\alpha} + \frac{1}{\beta}$

Q5.For what value of 'a' do the pair of linear equations $ax + y = a^2$ and $x + ay = 1$, have (a) no solution (b) infinitely many solutions.

Q6.Aman takes two hour more than Nishant to cover a distance of 30km. If Aman doubles his speed, he would have taken 1 hour less than Nishant. Find speed of Aman and Nishant.

Q7 Find a relation between X and Y if the area of triangle formed by the points (x,y),(1,2) and (7,0) is 7 square unit.

Q8 IF the points A (1,-2),B(-4,-3),C(x,-2) and D(2,3) form a parallelogram, find the value of x.

Q9 Prove that $(\text{SinA}+\text{CosecA})^2 + (\text{CosA}+\text{SecA})^2 = 7+\tan^2 A+\text{Cot}^2 A$

Q10IF SecA+TanA=P, prove that SinA=$P^2$-1/$P^2$-1

Q11 Two men standing on either side of a cliff 90m high, observes the angle of elevation of the top of the cliff to be 30and 60respectively.Find the distance between the two men.

Q12Two Poles of equal heights are standing opposite to each other on either side of the road, which is 80m wide. From a point between them on the road, the angle

Of elevation of the top of the poles are 60and 30.Find the height of the poles and distance of the point from the poles.

Q13.In a right angled triangle ABC, angle C=$90^0$ and D, E and F are three points on BC such that they divide it in four equal parts. Prove that:

$8(AF^2+AD^2) = 11\ AC^2 + 5AB^2$

Q14. Prove that the ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides.

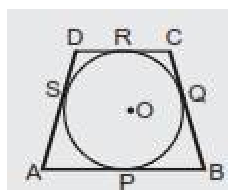
Q15. The sum of n terms of an A.P. is $(2n+3n^2)$.Determine the A.P. and find its $r^{th}$ term.

Q16. If the sum of first m terms of an A.P. is n and the sum of first n terms is m, then show that the sum of its (m+n) terms is – (m+n).

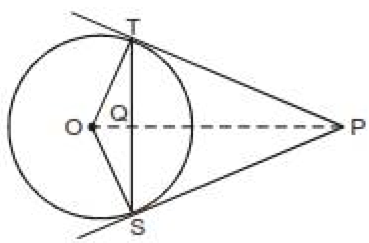
Q17. Find the value of p for which the following quadratic equation has two equal roots: $(p-12)\ x^2 +2(p-12)\ x +1 =0$.

Q18. A sailor can row a boat 8 km downstream and return back to the starting point in 1 hour 40 mints. If the speed of the stream is 2 km per hour, find the speed of the boat in still water.

Q19 In following Fig, a quadrilateral ABCD is drawn to circumscribe a circle, with centre 0, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that. AB + CD = BC + DA.




Q20 In Fig, from an external point P, two tangents PT and PS are drawn to a circle with centre O and Radiusr. If OP = 2r, show that □OTS = □OST= 30°.




Q21 In Fig. 7, two equal circles, with centre's O and O', touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre 0, at the point C. O'D is perpendicular to AC. Find the value of DO/CO.

Q22. Draw a circle of radius 6 cm. from a point 10cm away from its centre, construct the pair of tangents to the circle and measure their lengths.
Q23. In fig.4, 0 is the centre of a circle such that diameter AB= 13 cm and AC = 12 cm. BC is joined. FIND the area of the shaded region. (Take □ = 3.14)
Q 24 – One says "Give me a hundred, friend! I shall then become twice as rich as you are". The other replies," If you give me 10, I shall be 6 time as rich as you". How much each has?

Q 25 A number x is selected at random from the numbers 1, 2,3 and 4. Another number y is selected at random from the number 1, 4, 9 and 16. Find the probability that the product of x & y is less than 16

# Answer Key

**Solution 1**.Let $5-\frac{2}{7}\sqrt{3}$ is a rational

a-5 = $-\frac{2}{7}\sqrt{3}$

(a-5) $\times\frac{-7}{2}=\sqrt{3}$

$\frac{-7a}{2}+\frac{35}{2}=\sqrt{3}$

$\frac{-7a+35}{2}=\sqrt{3}$

Now a is rational number,

$\therefore\ \frac{-7a+35}{2}$ is a rational

Or $\sqrt{3}$ is a rational number

Which is false, $\because\sqrt{3}$ is not a rational number.

Hence our supposition is wrong

$\therefore$ **$5-\frac{2}{7}\sqrt{3}$ is not a rational number.**

**Solution2.** a = 210, b = 55, a $> b$

Using Euclid's division Lemma to 210 and 55, we get

210 = 55× 3 + 45

Now remainder 45$\neq 0$, apply Euclid's division lemma to 55 and 45

Here a = 55, b = 45

55 = 45 ×1 + 5

Again remainder 10$\neq$ 0, apply Euclid's division lemma to 45 and 10

here a= 45, b= 10

$45 = 10\times4 + 5$

Once again remainder 5$\neq$ 0, apply Euclid's division lemma to 10 and 5

Here, a= 10, b= 5

$10 = 5\times 2 +0$

Here remainder = 0

$\therefore$ HCF = 5

Now $5 = 210\times 5 + 55y$

$55y = 5- 1050= -1045$

**y= -19**

**Solution3.** As 1 is zero P(x)

$P(1) = 0= a^2$

$a(1)^2 - 3 (a-1) \times1-1=0$

$a- 3a+ 3- 1=0$

$-2a +2= 0$

$-2a= -2$

$\therefore$ **a=1**

**Solution4.** $P(x) = 3x^2+ 2x -6$

Here, a=3, b=2, c=-6

$\alpha + \beta = \frac{-b}{a} = \frac{-2}{3}, \ \alpha\beta = \frac{c}{a} = \frac{-6}{3} = -2$

$(a). \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2 \alpha\beta = (\frac{-2}{3})^2 - 2(-2)= \frac{4}{9} + 4= \frac{4+36}{9} = \frac{40}{9}$

$(b).\ \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\,\alpha\beta(\alpha + \beta) = (\frac{-2}{3})^3 - 3(-2)(\frac{-2}{3}) = \frac{-8}{27} - \frac{-12}{3} = \frac{-126}{27}$

(c) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-2}{3}}{-2} = \frac{-2}{3} \times \frac{-1}{2} = \frac{1}{3}$

**Solution5.** $ax + y = a^2$ ; $x + ay = 1$

Here, $a_1 = a$, $b_1 = 1$, $c_1 = a^2$

. $a_2 = 1$, $b_2 = a$, $c_2 = 1$

(a) For no solution. $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$

$$\frac{a}{1} = \frac{1}{a} \neq \frac{a^2}{1}$$

Or $a^2 = 1$

Or $a = \pm\sqrt{1}$

**Or a = +1, -1**

Also, $\frac{1}{a} \neq \frac{a^2}{1}$

Or, $a^3 \neq 1$

Or, $a^3 \neq 1^3$

**Or, a≠ 1**

(b) For infinitely many solution

$\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$

or $\frac{a}{1} = \frac{1}{a} = \frac{a^2}{1}$

$\frac{a}{1} = \frac{1}{a}$

Or $a^2 = 1$

Or $a = \pm\sqrt{1}$

**Or a = +1, -1**

Also, $\frac{1}{a} = \frac{a^2}{1}$

Or, $a^3 = 1$

Or, $a^3 = 1^3$

**Or, a= 1**

**Hence a=1**

**Solution6.**

Let speed of Aman is = x km/h

And speed of Nishant = y km/h

Distance = 30 km

Time taken by Aman = $T_1 = \frac{30}{x}$ h

Time taken by Nishant = $T_2 = \frac{30}{y}$ h

According to 1st condition,

$\frac{30}{x} - \frac{30}{y} = 2$ ----------- (1)

2nd condition

Speed of Aman = 2x km/h

Time taken by Aman = $\frac{30}{2x}$ h

Then, $\frac{30}{2x} - \frac{30}{y} = -1$ ----------- (2)

Let $\frac{1}{x}$ = u, and $\frac{1}{y}$ = v

From eq. 1, 30u – 30v = 2 ---------------- (3)

From eq. 2, 15u – 30v = -1 ----------------- (4)

Subtract (4) from (3)

$30u - 30v = 2$

$+15u - 30v = -1$

$- \quad + \quad +$

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$15u = 3$

Or $u = \frac{1}{5}$

Put u in (3) , we get

$30 \left(\frac{1}{5}\right) - 30v = 2$

Or, $-30v = 2-6$

Or, $-30v = -4$

Or, $v = \frac{2}{15}$

Now $u = \frac{1}{5}$

$\frac{1}{x} = \frac{1}{5}$

Or x = 5km/h

$v = \frac{2}{15}$

$\frac{1}{y} = \frac{2}{15}$

Or $y = \frac{15}{2} = 7.5$ km/h

Hence speed of Aman = 5 km/h and speed of Nishant is 7.5 km/h

**Solution 7** Area of triangle=7square unit

$1/2[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=7$

$1/2[x(2-0)+1(o-y)+7(y-2)]=7$

$[2x-y+7y-14]=14$

$2x+6y-14=14$

$2x+6y-28=0$

$x+3y-14=0$

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**Solution 8)** Mid point of BD=$\frac{2-4}{2}$ , $\frac{3-3}{2}$ =(-1,0)

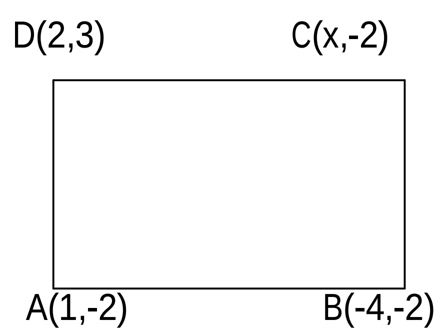
Mid point of AC= $\{\frac{x+1}{2}, \frac{2-2}{2}\} = (\frac{x+1}{2},0)$

Mid point of BD=Mid point of AC

$(-1,0) = (\frac{X+1}{2},0)$

$\frac{X+1}{2} = -1$

X= -3

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**Solution 9)** $Sin^2A+Cosec^2A+2SinA.CosecA+Cos^2A+Sec^2A+2CosA.SecA$

$1+2+2+Cosec^2A+Sec^2A$

$5+1+Cot^2A+1+Tan^2A$

$7+Tan^2A+Cot^2A.$

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**Solution 10)** SecA+TanA=P

$\frac{1}{COSA} + \frac{SinA}{COSA}$ =P , 1+SinA=PcosA (Squaring both side)

Mid point of AC= { $\frac{x+1}{2}$ , $\frac{2-2}{2}$}= ( $\frac{x+1}{2}$,0)

C(x,-2)

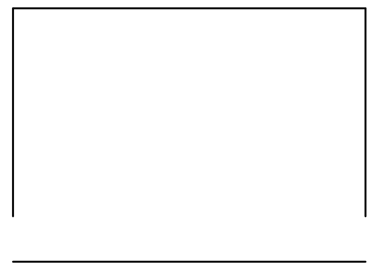
Mid point of BD=Mid point of AC

(-1,0) = ($\frac{X+1}{2}$,0)

$\frac{X+1}{2}$ = -1

B(-4,-2)

X= -3

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$Sin^2A+Cosec^2A+2SinA.CosecA+Cos^2A+Sec^2A+2CosA.SecA$

$1+2+2+Cosec^2A+Sec^2A$

$5+1+Cot^2A+1+Tan^2A$

$7+Tan^2A+Cot^2A$.

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SecA+TanA=P

$\frac{1}{COSA}$ + $\frac{SinA}{COSA}$ =P , 1+SinA=PcosA (Squaring both side)

$(1+sinA)^2 = P^2cos^2A$ ,$(1+SinA)^2=P^2(1+SinA)(1-SinA)$

$1+SinA=P^2-P^2SinA$ , $SinA+P^2SinA=P^2-1$, $SinA(1+p^2)=p^2-1$

SinA=$\frac{P2-1}{1+P2}$

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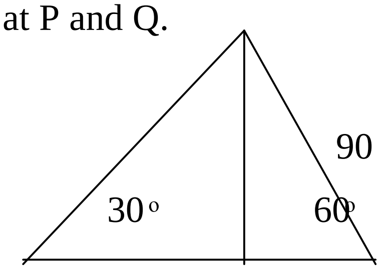
**Solution11**) Let AB be the cliff and two men are standing at P and Q.

AB=90 , < APB=30° and <AQB=60 °

In right triangle ABP, $\frac{AP}{BP}$ = tan 30°

. $\frac{90}{BP} = \frac{1}{\sqrt{3}}$ ,BP=90√3m




In right triangle ABQ, $\frac{AB}{BQ}$=Tan 60°, $\frac{90}{BQ}$ =√3, BQ=$\frac{90}{\sqrt{3}}$m

PQ=BP+BQ=90√3+$\frac{90}{\sqrt{3}}$=$\frac{360}{\sqrt{3}}$ =120√3

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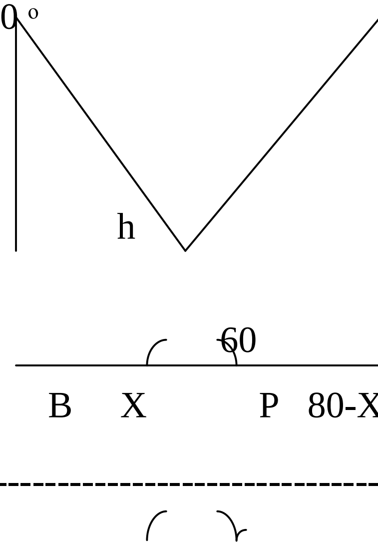
**Solution12)** In right triangle ABP, $\frac{AB}{BP}$ =tan 60 $^o$ A C




$\frac{h}{x}=\sqrt{3}$ , h=$\sqrt{3}x$ ,In right triangle CDP

$\frac{\sqrt{3}x}{80-x}=\frac{1}{\sqrt{3}}$

h

3x=80-x, 4x=80, x=20, 80-x,80-20=60.

h=$\sqrt{3x}$, h=20$\sqrt{3}$

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**Solution 13**Proof: In ΔABC, angle C =$90^0$

$AF^2 = AC^2+FC^2$

$AF^2=AC^2 + (BC/4)^2$

$16AF^2=16AC^2+BC^2$...........(1)

In Δ ACD,

$AD^2=AC^2+CD^2$

$16AD^2 =16AC^2+9BC^2$.........(2)

$8(AC^2+AD^2)=11AC^2 +5AB^2$

**Solution 14**By similarity, ar(ABC) = AB2 =BC2 =AC2.

Ar(DEF) DE2 EF2 DF2

**Solution 15**A.P. is 5,11,17 .......... and rth term is 6r-1.

**Solution 16** Let the A.P is a,a+d,a+2d,.....

m/2(2a+(m-1)d)=n

n/2(2a+(n-1)d)=m

after subtracting,2a+(m+n-1)d=-2

$S_{m+n}$ =m+n/2(2a=(m+n-1)d)

= m+n/2(-2) = -(m+n).

**Solution 17**For equal roots, D=$b^2$-4ac

P= 14.

**Solution 18** Let the speed of the boat in still water be x km/hr.

8/x+2 + 8/x-2 = 100/60

48x = $5x^2$ -20.

$5x^2$ -50x+2x-20=0.

(x-10)(5x+2)=0

X=10 or x= -2/5.

So,speed of the boat in atill water is 10 km /hr.

**Solution 19**. As we know that tangent from same external points are equal

 SD = DR ...(1)

CQ = CR ...(2)

QB = BP ...(3)

AS = AP ...(4)

Adding equation (1), (2), (3) & (4)

SD + CQ + QB + AS = DR + CR + BP + AP

AD + BC = AB + DC Hence proved

**Solution 20** In OTP

OT = r, OP = 2r [Given]

 OTP = 90° [radius is perpendicular to tangent at the pair of contact]

Let □TPO = □

□ sin□ = $\frac{OT}{OP} = \frac{1}{2}$

□ □□= 30°

□ In □TOP □TOP = 60° [By angle sum property]

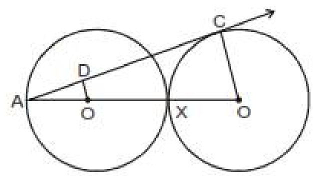
□□TOP = □□SOP [As □'s are congruent]

□□□□SOP is also 60°

□□□□TOS = 120° In □OTS as OT = OS □ [□□OST = □□OTS]

□□OTS + □□OST + □□SOT = 180 □ 2 □□OST + 120 = 180°

□□□□OTS + □□OST = 30°




**Solution 21** Let AO'= OX'= XO = r

□ Radius is always perpendicular to tangent, □ □ACO = 90°

In □ADO and □ACO

□DAO' = □CAO [Common]

□ADO' = □ACO [each 90°]

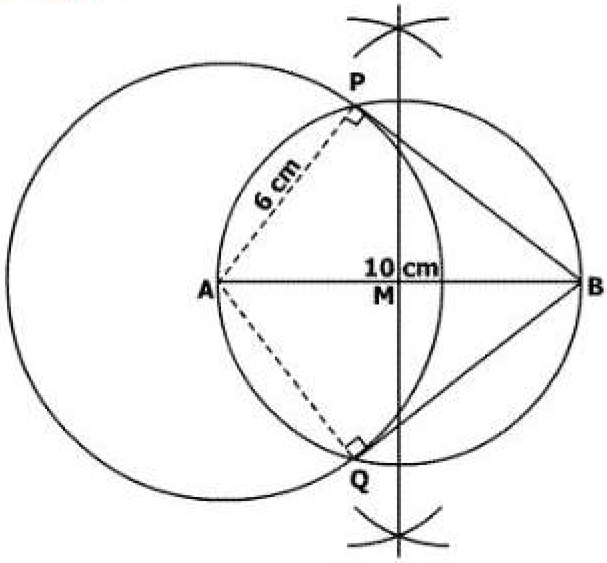
□ By AA similarity criteria

□ADO' ~ □ACO

Therefore $\frac{DO}{CO} = \frac{R}{3R} = \frac{1}{3}$

Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

Solution:

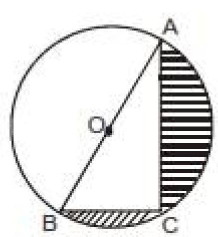



**Solution 22.**

**Steps of construction**

1. Draw a line segment of length AB = 10 cm. Bisect AB by constructing a perpendicular
   bisector of AB. Let M be the mid-point of AB.
2. With M as centre and AM as radius, draw a circle. Let it intersect the given circle at the points P and Q.
3. Join PB and QB. Thus PB and QB are the required two tangents.

**Solution 23**



$AB^2 = BC^2 + AC^2$

$\Rightarrow 169 = BC^2 + 144$

$25 = BC^2$

$BC = 5$

Area of shaded region = Area of semicircle – Area of $\Delta ABC$

$= \frac{\pi r^2}{2} - \frac{1}{2} \times BC \times AC$

$= \frac{1}{2}[3.14 \times \frac{13}{2} \times \frac{13}{2}] - (5 \times 12)$

$= \frac{1}{2}(132.665 - 60)$

$= 36.3325\ cm^2$

**Solution 24** Let the amount of their respective capital be Rs x & Rs y
Therefore according to given condition

$x+100 = 2(y-100)$

$x-2y = -300$ ____(1)

and $6(x-10) = y+10$

or $6x-y=70$ ____(2)

$$\begin{array}{rcl} x \ -2y &=& -300 \\ +12x \ -2y &=& 140 \\ \underline{- \quad\quad + } && \underline{-} \\ -11x &=& -440 \end{array}$$

Or $x=40$

Similarly $y = 170$

**Solution25** :- $P(x) = ¼$

$P(y) = ¼$

$P(x).P(y) = ¼ \times ¼ = 1/16$ which is less than 16