DELHI PUBLIC SCHOOL JAMMU

QUESTION BANK
SESSION: 2018-19
CLASS: X
SUBJECT: MATHEMATICS

Topics:

1. **Constructions**
2. **Area related to Circle**
3. **Arithmetic Progression**
4. **Statistics**
5. **Probability**
6. **Surface Area and Volume**

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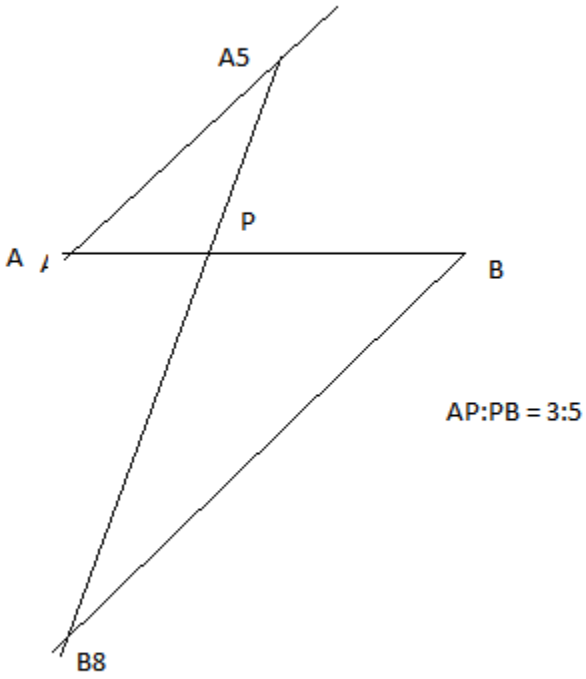
TOPIC: CONSTRUCTION

Q1 Divide a line segment of length 7cm in the ratio 5:8. Also justify your construction.

Sol:- Diagram



Justification:

In ΔAPA5 and ΔBPB8

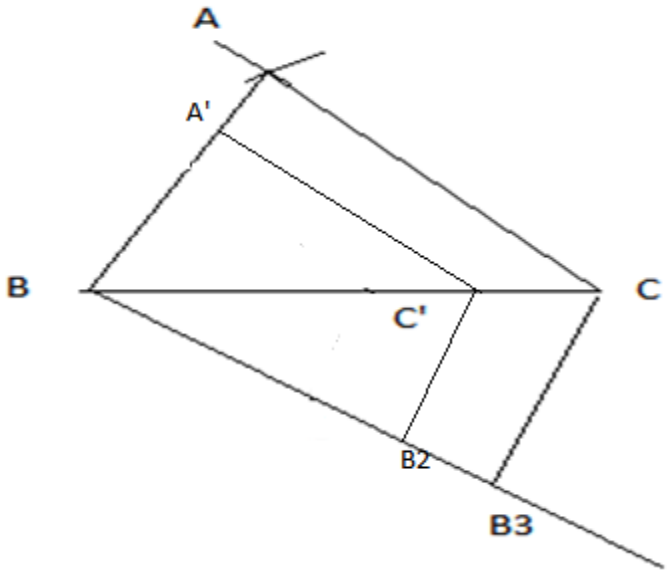
$\angle A = \angle B$ **(ail. Int angles)**

$\angle APA5 = \angle BPB8$ **(vert. Opp angles)**

∴ ΔAPA5 ~ ΔBPB8 (AA rule)

$\frac{AP}{PB} = \frac{AA5}{AA8} = \frac{5}{8}$

Q 2. Construct a triangle of sides 4cm, 6cm and 5cm, and then construct a similar triangle whose sides are 2/3 of corresponding sides of the first triangle.



Justification:- ΔBC'B2 and ΔBCB3

$\angle B = \angle B$ **(common)**

$\angle BB2C' = \angle BB3C$ **(corros. Angles)**

∴ ΔBC'B2 ~ ΔBCB3 (AA rule)

$\frac{BC'}{BC} = \frac{BB2}{BB3} = \frac{2}{3}$

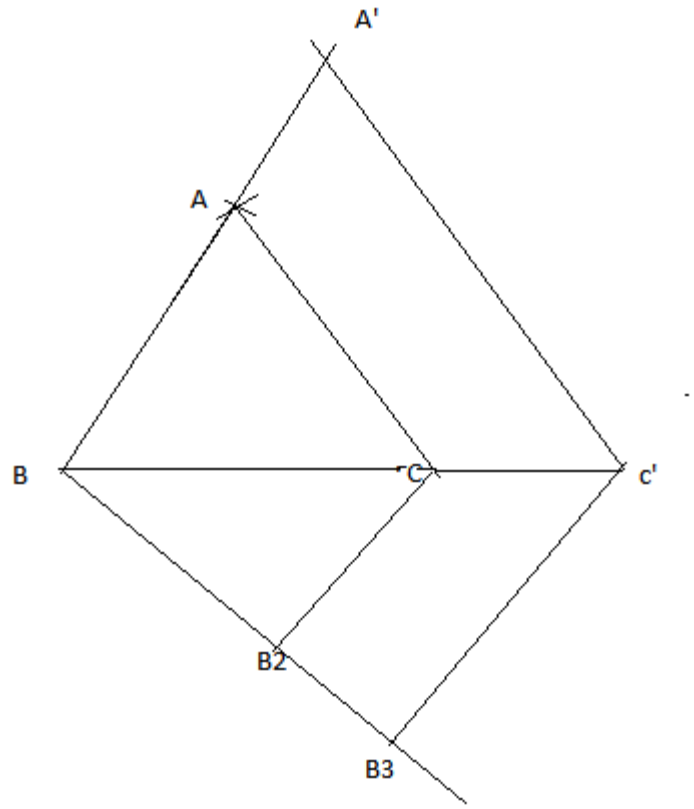
Similarly $\Delta A'BC' \sim \Delta ABC$

Or $\frac{BC'}{BC} = \frac{A'B}{AB} = \frac{2}{3}$

$\Delta A'BC' = 2/3\ \Delta ABC$

Q3 Construct a triangle similar to triangle ABC with BC=5cm, AB=AC= 6cm, such that it is 3/2 of given triangle.

Sol.

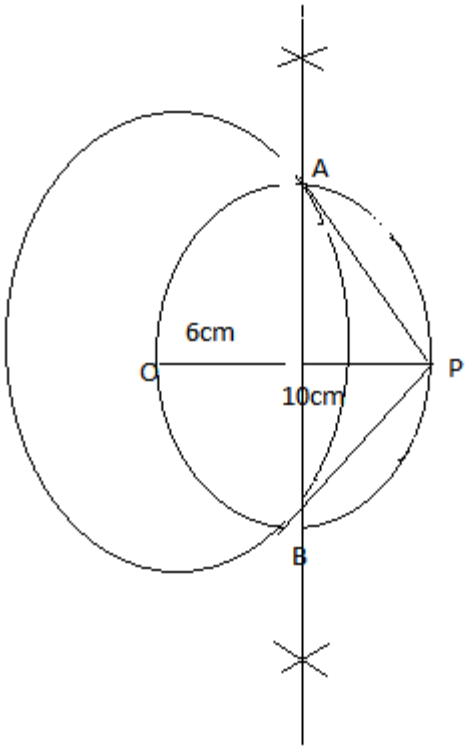


Justification:- ΔBC'B3 and ΔBCB2

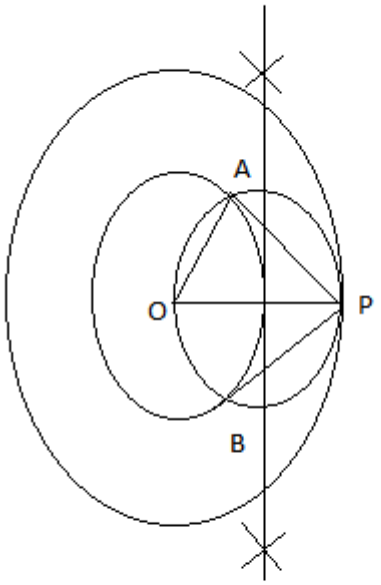
$\angle B = \angle B$ **(common)**

$\angle BB3C' = \angle BB2C$ **(corros. Angles)**
∴ ΔBC'B2 ~ ΔBCB3 (AA rule)
$\frac{BC'}{BC} = \frac{BB3}{BB2} = \frac{3}{2}$
Similarly ΔA'BC' ~ ΔABC
Or $\frac{BC'}{BC} = \frac{A'B}{AB} = \frac{3}{2}$
Δ A'BC' = 3/2 ΔABC

Q4. Draw a circle of radius 6cm. from a point 10cm away from centre, construct a pair of common tangents to the circle.



Justification;
Join OA
OP is diameter of circle through O and P
So, $\angle OAP = 90°$ (angle in the semicircle)
Or PA ⊥ OA
But perpendicular at point of contact.i.e at end of radius is tangent to the circle
∴ PA is tangent to the circle. Similarly PB is tangent to the circle.
Q5. Draw two concentric circles of radii 3cm and 5cm. Now from external circle draw tangent to the internal circle.
Sol.



Justification;
Join OA
OP is diameter of circle through O and P
So, $\angle OAP = 90°$ (angle in the semicircle)
Or PA $\perp$ OA
But perpendicular at point of contact.i.e at end of radius is tangent to the circle
$\therefore$ PA is tangent to the circle. Similarly PB is tangent to the circle.

TOPIC: AREAS RELATED TO CIRCLES

Q1. The cost of fencing a circular field at the rate of 36 per m is 11880. The field is to be ploughed at the rate of 0.60 per m^2. Find the cost of ploughing the field. [Take, π = 22/7]

Sol. Length of the fence of a circular field $= \frac{Total\ cost}{Rate} = \frac{11880}{36} = 330\ m$

So, the circumference of the field is 330 m.

Let r be the radius of the field, Then

$\therefore \quad 2\pi r = 330$ [$\because$ corcumference of circular field = $2\pi r$]

$\Longrightarrow \ 2 \times \frac{22}{7} \times r = 330$

$\Longrightarrow \ 2 \times \frac{330 \times 7}{2 \times 22} = 52.5\ m$

So, the radius of the field is 52.5 m.

Now, area of the field $\Longrightarrow \ \pi r^2 \times \frac{22}{7} \times 52.5 \times 52.5 \ = \ 22 \times 7.5 \times 52.5\ m^2$

Given, cost of ploughing 1 m^2 of the field = ₹ 0.60

So, total cost of ploughing the field $= 0.6 \times 22 \times 7.5 \times 52.5 = \ ₹\ 5197.5$

Q2. The radius of the wheel of a bus is 25 cm. If the speed of the bus is 33 km/h, then how many revolutions with the wheel make in 1 min?

Sol. In 1 h, distance covered by wheel = 33 km

In 1 min, distance covered by wheel = $\frac{33 \times 1000}{60}\ m = 550\ m$

Now, number of revolutions made in 1 min = $\frac{Distance\ vovered\ by\ wheel}{Circumference\ of\ the\ wheel}$

$= \frac{550}{2\times\frac{22}{7}\times\frac{25}{100}}$ $\quad [\because 25cm = \frac{25}{100}m]$

$= \frac{550\times7\times100}{2\times22\times25} = 350$

Q3. The short and long hands of a clock are 6 cm and 8 cm long, respectively. Find the sum of the distance travelled by their tips in 1 day. [Take, π = 22/7]

Sol. In 1 day, i.e. 24 hr, short (hour) hand of the clock make 2 revolutions and long (minute) hand make 24 revolutions.

In 1 revolutions, distance travelled by tip of hour hand

= Circumference of circle of radius 6 cm = $2\times\frac{22}{7}\times6$

In 1 revolutions, distance travelled by tip of minute hand

= Circumference of circle of radius 8 cm = $2\times\frac{22}{7}\times8$

∴ Sum of distance travelled by tips of both hand in 1 day

$= 2\times2\times\frac{22}{7}\times6 + 24\times2\times\frac{22}{7}\times8$

$= 2\times\frac{22}{7}(12+192) == 2\times\frac{22}{7}\times204$

= 1282.29 cm (approx).

Q4. Find the area of the segment AYB, if radius of the circle is 28 cm and $\angle AOB = 90^\circ$ **[Take, π = 22/7].**

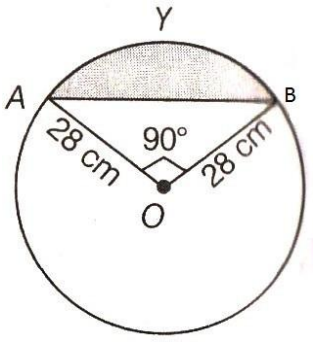
Sol. Given, r = 28 cm and $\angle AOB = 90^\circ$

∴ Area of the sector OAYB = $\frac{90^\circ}{360^\circ}\times\frac{22}{7}\times28\times28 = 616\ cm^2$ $\left[\because area\ of\ sector = \frac{\theta}{360^\circ}\times\pi r^2\right]$

Now, in ΔOAB, OA = OB = 28 cm, $\angle AOB = 90^\circ$

Draw OM ⊥ AB, then by RHS congruence, ΔAMO ≅ ΔBMO

So, M is the mid-point of AB. and $\angle AOM = \angle BOM = \frac{\angle AOB}{2} = \frac{90^\circ}{2} = 45^\circ$



In right angled ΔOMA, cos 45° = $\frac{OM}{OA}$

$\Longrightarrow \frac{1}{\sqrt{2}} = \frac{OM}{28} \Longrightarrow OM = \frac{28}{\sqrt{2}}$ $\quad \left[\because \cos 45^\circ = \frac{1}{\sqrt{2}}\right]$

Also, $\sin 45^\circ = \frac{AM}{OA}$

$\Longrightarrow \frac{1}{\sqrt{2}} = \frac{AM}{28} \Longrightarrow AM = \frac{28}{\sqrt{2}}cm$

$\therefore AB = 2AM = 2\times\frac{28}{\sqrt{2}}$

$= 2\times\frac{28}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = 28\sqrt{2}$ **cm**

Now, area of $\Delta\ OAB = \frac{1}{2}AB\times OM = \frac{1}{2}\times28\sqrt{2}\times\frac{28}{\sqrt{2}}$

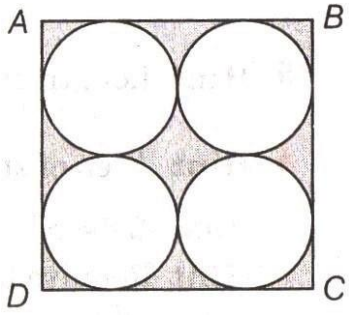
$$\left[\because area\ of\ triangle = \frac{1}{2}\times base\times height\right] = 392\ cm^2$$

Hence, area of segment AYB = Area of sector OAYB - Area of ΔOAB

$= 616 - 392 = 224\ cm^2$

Q5. Find the area of the shaded region in figure as shown below where ABCD is a square of side 12 cm

Sol. ∵**Area of square = (Side)^2**



∴ Area of square ABCD = 12 x 12 = 144 cm^2

∵ Diameter of each circle = $\frac{12}{2} = 6\ cm$ [here, lenght = 12 cm]

∴ Radius of each circle (r) = $\frac{6}{2} = 3\ cm$

Area of one circle = $\pi r^2 \frac{22}{7} \times 3 \times 3 = \frac{198}{7}\ cm^2$

Therefore, area of four circles = $4 \times \frac{198}{7} = \frac{792}{7} cm^2$

Hence, area of the shaded region = area of square - Area of four circles

$= 144 - \frac{792}{7} = \frac{1008-792}{7} = \frac{216}{7} = 30.86\ cm^2$

Q6. A memento is made as shown in the figure. Its base PBCR is silver plated from the front side at the rate of ₹ 20 per cm^2. Find the total cost of the silver plating. [Take, π = 22/7].

Sol. From the figure, we have AB = 7 + 3 = 10 cm

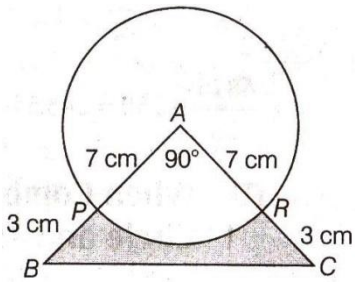
Ac = 7 + 3 = 10 cm

Also radius of the circle = AP = AR = 7 cm.

∴ Area of right angled ΔBAC = $\frac{1}{2} \times AB \times AC = \frac{1}{2} \times 10 \times 10 = 50\ cm^2$

Area of the sector APR = $\frac{90^\circ}{360^\circ} \times \pi r^2 = \frac{1}{4} \times \pi \times (7)^2$ $\left[\because area\ of\ sector = \frac{\theta}{360^\circ} \times \pi r^2\right]$

$= \frac{1}{4} \times \frac{22}{7} \times 49 = \frac{11 \times 7}{2} = 38.5\ cm^2$



Then, area of base PBCR (shaded) which is to be silver plated

= Area of the right angled ΔBAC - Area of the sector APR

= 50 - 38.5 = 11.5 cm^2

∴ Total cost of silver plating at the rate of ₹ 20 per cm^2

= ₹ 20 x 11.5 = ₹ 230

Q 7. Find the area of the shaded region in figure, if AC = 20 cm, AB = 15 cm and O is the centre of the circle. [Take, π = 22/7].

Sol. It is clear from the figure, BC is diameter of the circle

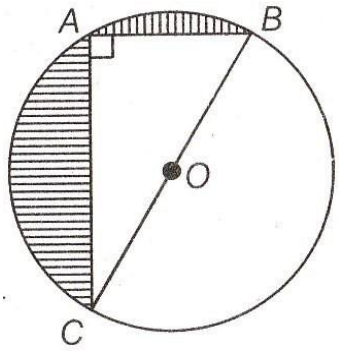
∴ $\angle BAC = 90^\circ$ [∵ angle in a semi-circle is a right angle]

Now, in right angle ΔBAC, (BC)^2 = AB^2 + AC^2 [using Pythagoras theorem]

∴ $BC = \sqrt{(15)^2 + (20)^2}$ [∵ $AC = 20\ cm, AB = 15\ cm$]

$\Longrightarrow BC = \sqrt{625} \implies BC = 25$

∴ radious of the circle, $r = \frac{25}{2} cm$



Now, area of shaded region = Area of semi-circle - Area of ΔABC

$= \frac{\pi r^2}{2} - \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - \frac{1}{2} \times 20 \times 15$

$= \frac{6875}{28} - 150 = 245.54 - 150 = 95.54\ cm^2$

Q8. The area of an equilateral ΔABC is 17320.5 cm^2. Which each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see he figure). Find the area of the shaded region. [Take, π = 3.14 and $\sqrt{3}$ = 1.73205]

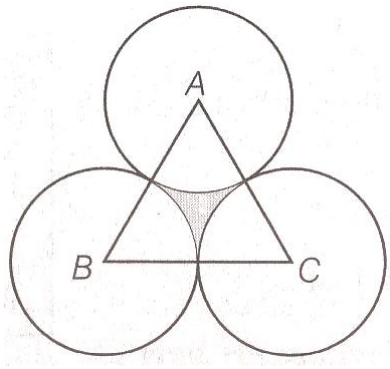
Sol. Let the side of an equilateral triangle be a.

∴ Area of an equilateral Δ ABC = $\frac{\sqrt{3}}{4}(a)^2$

But given, area of equilateral Δ ABC = 17320.5 cm^2

∴ $17320.5 = \frac{1.73205}{4}(a^2)$

⟹ $a^2 = 10000 \times 4 \Rightarrow a = 100 \times 2 = 200\ cm$ [taking positive square root]



Since, ΔABC is an equilateral triangle.

∴ $\angle A = \angle B = \angle C = 60^\circ$ Here, AB = a = 200 cm

∴ Radius of circle = $\frac{200}{2} = 100\ cm$ [∴ radius of circle = half the length of the side of the Δ ABC]

Area of sector of a circle = $\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times 3.14 \times (100)^2 = 5233.33\ cm^2$

Then, area of three equal sectors $= 3 \times 5233.33 = 15700\ cm^2$

∴ Area of required shaded region = Area of ΔABC - Area of three sectors

= 17320.5 - 15700 = 1620.5 cm^2

STATISTICS

Very short answer type questions:

Q1. The mean of 6, 6 + 2x, 5 and 8 + 3x is 20. Find the value of x.

Sol. ∵ Mean = $\frac{x_1 + x_2 + \ldots\ldots\ldots + x_n}{n}$

∵ $\frac{6+6+2x+5+8+3x}{4} = 20$ [∵ mean = 20]

⟹ 25 + 5x + 80

⟹ 5x = 80 - 25 ⟹ x = 11

Q2. If the mode of a data is 53 and mean is 33, then find its median.

Sol. We know that,

3 Median = Mode + 2 Mean

⟹ Median = $\frac{1}{3}$ Mode + $\frac{2}{3}$ Mean

= $\frac{1}{3}$ x 53 + $\frac{2}{3}$ x 33

= $\frac{53+66}{3} = \frac{119}{3}$

= 39.67

Q3. The mean of 20 observations is 12. If each observation is increased by 5, then find the new mean.

Sol. If each observation is increased by 5, then the required new mean is also increased by 5. Thus, the new mean is 17.

Q4. Find the median of the following values: 37, 31, 42, 43, 46, 25, 39, 43, 32.

Sol. For computing median, we first arrange the data in an ascending order.

25, 31, 32, 37, 39, 42, 43, 43, 46

Here n = 9 (the number of observations are odd).

$\therefore$ The median is the $\left(\frac{n+1}{2}\right)$th observation.

Median = $\left(\frac{9+1}{2}\right)$th observation in the arranged data

= 5th observation = 39

Short answer type questions:

Q5. The following table shows the heights (in cm) of 50 girls of class X of a school.

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total
Number of Girls	2	8	12	20	8	50

Find the mean of the above data by step-deviation method.

Sol. Let assumed mean, A = 145 and h = 10. Table for step deviation is given below

Height (in cm)	Number of Girls (f_i)	Class Marks (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
120-130	2	125	-2	-4
130-140	8	135	-1	-8
140-150	12	145 = A	0	0
150-160	20	155	1	20
160-170	8	165	2	16
Total	$\sum_i^f = 50$			$\sum_i^f u_i = 24$

Here, $\sum f_i$ = 5- and $\sum f_i u_i = 24$

$\therefore$ Mean, $\bar{x}$ = A + $\sum f_i u_i = 24$

= 145 + 4.8 = 149.8

Q6. Calculate the median for the following data.

Class Interval	130-139	140-149	150-159	160-169	170-179	180-189	190-199
Frequency	4	9	18	28	24	10	7

Here inclusive classes are given. Therefore, for median determination, inclusive class limits need to be converted into continuous class as shown in the table given below.

Given Classes	Class Boundaries	Frequency	c.f.
130-139	129.5 - 139.5	4	4
140-149	139.5 - 149.5	9	13
150-159	149.5 - 159.5	18	31
160-169	159.5 - 169.5	28	59
170-179	169.5 - 179.5	24	83
180-189	179.5 - 189.5	10	93

190-199	189.5 - 199.5	7	100

Here, n = 100.

So, $\frac{n}{2} = \frac{100}{2} = 50$

This observation lies in the class 159.5 - 169.5

$\therefore$ l = 159, $\frac{n}{2}$ = 50, f = 28, c.f. = 31 and h = 10.

$\because$ Median = $l + \frac{\frac{n}{2} - cf}{f} \times h$

= $159.5 + \frac{50-31}{28} \times 10$

= $159.5 + \frac{190}{28} = 159.5 + 6.79 = 166.29$

Q7. The following data gives the distribution of total household expenditure (in Rs.) of manual workers in a city.

Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Frequency	24	40	33	28	30	22	16	7

Find the mode from the above data.

Sol. We observe that the class 1500-2000 has the maximum frequency 40.

So, it is the modal class such that

l = 1500, h = 500, f_1 = 40, f_0 = 24 and f_2 = 33

$\therefore$ Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

= $1500 + \left(\frac{40-24}{80-24-33} \times 500\right)$

= $1500 + \frac{16}{23} \times 500$

= 1500 + 347.826 = 1847.825

Short answer type questions:

Q8. The mean of the following frequency table is 50 but the frequencies f_1 and f_2 in class intervals 20-40 and 60-80 are missing. Find the missing frequencies.

Class Interval	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Sol. Let assumed mean be A = 50 and h = 20.

Table for step deviation is given below.

Class Interval	Frequency (f_i)	Mil Value (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
0-20	17	10	-2	-34
20-40	f_1	30	-1	$-f_1$
40-60	32	50	0	0
60-80	f_2	70	1	f_2
80-100	19	90	2	38
Total	$N = \sum_i^f = 68 + f_1 + f_2$		$\sum f_i u_i = 4 - f_1 +$	

We have, $N = \sum f_i = 120$ (given)

$\Rightarrow$ $68 + f_1 + f_2 = 120$

$\Rightarrow$ $f_1 + f_2 = 52$ ..... (i)

But mean = 50 (given)

$\therefore$ $A + h\left(\frac{1}{N}\sum f_i u_i\right) = 50$

$\Rightarrow$ $50 + 20 \times \left(\frac{4 - f_1 - f_2}{120}\right) = 50$

$\Rightarrow$ $50 + \frac{4 - f_1 + f_2}{6} = 50$

$\Rightarrow$ $\frac{4 - f_1 + f_2}{6} = 0$

$\Rightarrow$ $4 - f_1 + f_2 = 0$

$\therefore$ $f_1 - f_2 = 4$ ..... (ii)

On solving equation (i) and (ii), we get $f_1 = 28$ and $f_2 = 24$

TOPIC-PROBABILITY

Q1 All the black face cards are removed from 52 cards . The remaining cards are well shuffled and then a card is drawn .Find the probability of (1) face card (2)red card (3)black card (4)king

Ans Black face card =6

Total no of cards =46

Probability of face card=6/46

2) probability of Red card=20/46

3) probability of black card=20/46

4) probability of king=2/46

Q2 Cards from 11to 60 are kept in the box.if a card drawn random from the box,find the prob.of(1)an odd no. (2)a perfect square no. (3) divisible by 5 (4)a prime no. less than 20.

Ans. Total cards=60-11+1=50

(1)Prob.of odd no.=11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47,49,51,53,55,57,59=25/50

(2)Prob. Of perfect square no.=16,25,36,49=4/50

(3)Prob. Of divisible by 5=15,20,25,30,35,40,45,50,55,60=10/50

(4)Prob.of prime no.less than 5=11,13,17,19=4/50

Q3A no. X is selected random from the no.s 1,4,9,16 and another no. Y is selected random from the no.s 1,2,3,4.Find the prob.that the value of XY is more than 16.

Ans Total no. of X =4

Total no. of Y=4

Product of X and Y =4x4=16

Product more than 16=(9x2),(9x3),(9x4),(16x2),(16x3),(16x4)=6
Prob.=6/16.

Q4 Two dice are thrown once together.Find the prob.(1)same no. on both dice(2)different no. on both dice(3)not prime no.(4)even no. on both dice.

Ans Total no. of outcomes=36
(1)Prob.of same no. on both dice=(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)=6/36.
(2)Prob.of different no. on both dice=36-6=30/36
(3)Prob.of not prime no.=21/36.
(4)Prob.of even no. on both dice=9/36.

Q5Three coins are tossed together.Find the prob. Of (1)exactly two heads(2)at least two heads(3)at least two tails.

Ans (HHH),(HHT),(HTH),(THH),(HTT),(THT),(TTH)(TTT) Total outcomes=8
(1)Prob.of exactly two heads=(HHT),(HTH),(THH)=3/8
(2)prob.of at least two heads=(HHH),(HHT),(HTH),(THH)=4/8
(3)Prob.of at least two tails=(HTT),(THT),(TTH),(TTT)=4/8.

Q6 A bag contains 14 balls of which X are white.If 6 more white balls are added to bag,the prob. Of white ball is1/2.Find the value of X.

Ans Total balls=14
No. of white balls=X , now 6 more balls added.
No. of white balls=x+6
Total balls in the bag=14+6=20
Acc. To question
X+6/20=1/2
2x+12=20
2x=8
X=4.

Q7 A letter of English alphabets is chosen at random.find the prob. Of vowels.

Ans Total outcomes=26
No.of vowels=(a,e,I,o,u)=5
Prob. Of vowels=5/26.
Q8 What is the prob. Of a sure event.
Ans one(1).
Q9 What is the prob.that a non leap year have 53 Sundays.
Ans No. of days=365
No. of possible outcomes=7
No.of favourable outcomes=1
Reqd. prob.=1/7.

Q10 Find the sum of prob. of all the events of an experiment.

Ans one(1)

TOPIC: SURFACE AREA AND VOLUME

Q1. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44cm, each bullet being 4cm in diameter.

Sol.: Let the total number of bullets be x.

Radius of a spherical bullet $= \frac{4}{2}\text{cm} = 2\text{cm}$

Now, Volume of a spherical bullet = $= \frac{4}{2}\pi \text{ x } (2)^3 \text{cm}^3 = \left(\frac{4}{3} \text{x} \frac{22}{7} \text{x } 8\right)\text{cm}^3$

$\therefore$ Volume of x spherical bullets = $= \left(\frac{4}{3} \text{x} \frac{22}{7} \text{x } 8 \text{ x } x\right)\text{cm}^3$

Volume of the solid cube = $(44)^3$ cm^3

Clearly, Volume of x spherical bullets = Volume of cube

$$\Rightarrow \quad \frac{4}{3} \text{x} \frac{22}{7} \text{x } 8 \text{ x } x = (44)^3$$

$$\Rightarrow \quad \frac{4}{3} \text{x} \frac{22}{7} \text{x } 8 \text{ x } x = 44 \text{ x } 44 \text{ x } 44$$

$$\Rightarrow \quad \text{x} = \frac{44 \text{ x } 44 \text{ x } 44 \text{ x } 3 \text{ x } 7}{4 \text{ x } 22 \text{ x } 8} = 2541$$

Hence, total number of spherical bullets = 2541

Q2. Water is being pumped out through a circular pipe whose internal diameter is 7 cm. If the flow of water is 72 cm per second, how many liters of water are being pumped out in one hour ?

Sol.: We have,

Radius of the circular pipe $= \frac{7}{2}$ cm

Clearly, Water column forms a cylinder of radius $\frac{7}{2}$ cm.

It is given that the water flows out at the rate of 72 cm/sec.

$\therefore$ Length of the water column flowing out in one second = 72 cm.

Volume of the water flowing out per second

= Volume of the cylinder of radius $\frac{7}{2}$ cm and length 72 cm.

$$= \pi \text{ x} \left(\frac{7}{2}\right)^2 \text{ x } 72 \text{ cm}^3$$

$$= \pi \text{ x} \frac{7}{2} \text{x} \frac{7}{2} \text{ x } 72 \text{ cm}^3 = \frac{22}{7} \text{x} \frac{7}{2} \text{x} \frac{7}{2} \text{x } 72 \text{ cm}^3 = 2772 \text{cm}^3$$

$\therefore$ Volume of the water flowing out in one hour

= (2772 x 3600) cm^3 [$\because$ 1 Hr = 3600 sec.]

= 9979200 cm^3

$$= \frac{9979200}{1000} \text{litres} = 9979.2 \text{ litres}$$

Hence, 9979.2 liters of water flows out per hour.

Q3. Water in a canal, 30 dm wide and 12 dm deep is flowing with velocity of 10 km/hr. How much area will it irrigate in 30 minutes, if 8 cm of standing water is required for irrigation ?

Sol. We have, [CBSE 2014]

Width of the canal = 30 cm = 300 cm = 3 m

Depth of the canal = 12 dm = 120 cm = 1.2 m.

It is given that the water is flowing with velocity 10 km/hr. Therefore,

Length of the water column formed in $\frac{1}{2}$ hour = 5 km = 5000 m

$\therefore$ Volume of the water flowing in $\frac{1}{2}$ hour

= Volume of the cuboid of length 5000 m, width 3 m and depth 1.2 m

$\Rightarrow$ Volume of the water following in ½ hour

= 5000 x 3 x 1.2 m^3 = 18000 m^3

Suppose x m^2 area is irrigated in $\frac{1}{2}$ hour. Then,

$$x \text{ x } \frac{8}{100} = 18000$$

$$\Rightarrow \quad \text{x} = \frac{1800000}{8} \text{m}^2$$

$\Rightarrow$ x = 225000 m^2

Hence, the canal irrigates 225000 m^2 area in $\frac{1}{2}$ hour

Q4. A copper wire 3 mm in diameter is wound about a cylinder whose length is 1.2 m, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of the copper wire to be 8.88 gram per cm.

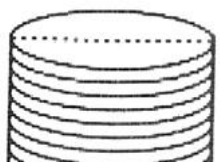
[NCERT]

Sol. We have,

d = Diameter of copper wire = 3 mm = $\frac{3}{10}$ cm

h = Height (length) of the cylinder = 1.2 m = 120 cm.

$\therefore$ Number of rounds taken by the wire to cover the



curved surface of the cylinder = $\frac{h}{d} = \frac{\frac{120}{3}}{10} = 400$

Length of wire used in taking one round = $2\pi r$

= (2 x 3.14 x 5) cm = 31.4 cm

$\therefore$ Total length of wire used in covering the curved surface of the cylinder

= (31.4 x 400) cm = 12560 cm = 125.6 m

Mass of the wire = Length x Density = 12560 x 8.88 gram = 111532.8 gm

= 111.5328 kg = 111.533 kg

Q5. The largest sphere is carved out of a cube of a side 7 cm. Find the volume of the sphere.

Sol. The diameter of the largest sphere which can be carved out of a cube of side 7 cm is 7 cm.

$\therefore$ Radius of the sphere = r = $\frac{7}{2}$ cm

Hence,

Volume of the sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow$ Volume of the sphere = $\frac{4}{3} x \frac{22}{7} x \left(\frac{7}{2}\right)^3 cm^3$

$\Rightarrow$ Volume of the sphere $\frac{4}{3} x \frac{22}{7} x \frac{343}{8} cm^3 = 179.66\, cm^3$

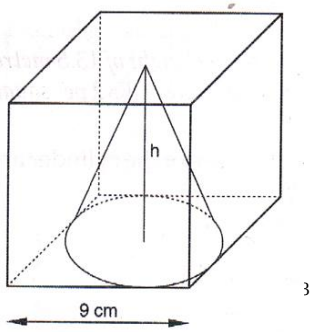
Q6. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9cm.

Sol. The base of the largest right circular cone that can be the circle inscribed in a face of the cube and its height will be equal to an edge of the cube.

$\therefore$ r = Radius of the base of the cone = $\frac{9}{2}$ cm [$\because$ edge = 9 cm]

H = Height of cone = 9 cm

Hence, Volume of the cone = $\frac{1}{3}\pi r^2 h$



$\Rightarrow$ Volume of the cone

$\Rightarrow$ Volume of the cone

Q7. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere as shown in Fig. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find its external surface area.

Sol. We have

r_1 = Radius of the lower end of the frustum = 1 cm

r_2 = Radius of the upper end of the frustum = 2.5 cm

h = Height of the frustum = 6 cm

l = Slant height of the frustum

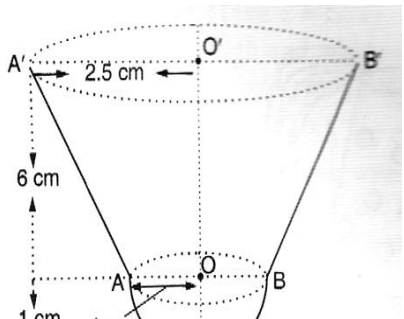
$l = \sqrt{h^2 + (r_2 - r_1)^2}$

$\Rightarrow$ $l = \sqrt{36 + (25-1)^2} = \sqrt{36.25 cm} = 6.18 cm$

$\therefore$ External surface area of shuttle cock

= Curved surface area of the frustum + Surface area of

$= \pi(r_2 - r_1)\, l + 2\pi r_1^2$



$= \{\pi (1+2.5) \text{ x } 6.18 + 2 \text{ x } \pi \text{ x } 1^2\} \text{cm}^2$

$= = \left\{\frac{22}{7} \text{ x } 3.5 \text{ x } 6.18 + 2 \text{ x } \frac{22}{7}\right\} \text{cm}^2 = (67.98 + 6.28) \text{cm}^2 = 74.26 \text{ cm}^2$

Q8. Find the length of the longest rod that can be place in a room 12m x 9m x 8m.

Sol. Length of the longest rod = Diagonal of the room

$= \sqrt{a^2 + b^2 + c^2} = \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17\text{m}$

Q9. A cylinder and a cone have equal heights and equal radii of their bases. If their curved surface areas are in the ratio 8 : 5. Show that the ratio of a radius to height of each is 3 : 4.

Sol. $\frac{\text{Curved surface area of cylinder}}{\text{Curved surface area of cone}} = \frac{2\pi rh}{\pi rl} = \frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}}$

$\frac{8}{5} = \frac{2h}{\sqrt{r^2 + h^2}}$

$\Rightarrow \quad \frac{64}{25} = \frac{4h^2}{r^2 + h^2} \quad \Rightarrow \quad 64r^2 + 64h^2 = 100h^2$

$\Rightarrow \quad 64r^2 = 100h^2 - 64h^2 \quad \Rightarrow \quad 64r^2 = 36h^2$

$\Rightarrow \quad \frac{r^2}{h^2} = \frac{36}{64} = \frac{9}{16} \quad \Rightarrow \quad \frac{r}{h} = \frac{3}{4}$

$\therefore$ **r : h = 3 : 4**

Q10. 30 Circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find :

i) the total surface area ii) volume of the cylinder so formed.

[NCERT Exemplar]

Sol. Height of the cylinder formed (h) = 30 x 3 = 90 cm

Radius of the base of the cylinder formed (r) = 14 cm

i) Total surface area of the cylinder $= 2\pi r(r+4)$

$= 2 \text{ x } \frac{22}{7} \text{ x } 14 (14 + 90)$

$= 2 \text{ x } \frac{22}{7} \text{ x } 14 \text{ x } 104 = 9152 \text{ cm}^2$

ii) Volume of the cylinder formed $= \pi r^2 h$

$= \frac{22}{7} \text{ x } 14 \text{ x } 14 \text{ x } 90 = 55440 \text{ cm}^3$

Q11. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilometers of water. Water is pumped into the tank to fill its capacity. Calculate the volume of water pumped into the tank.

[NCERT Exemplar]

Sol. Internal radius of the hemispherical tank (r) $= \frac{14}{2} \text{m} = 7 \text{m}$

Capacity of the tank = $\frac{2}{3}\pi r^3 = \frac{2}{3} \text{ x } \frac{22}{7} \text{ x } 7 \text{ x } 7 \text{ x } 7$

= 718.67 m^3 = 718.67 kilolitres

Volume of water pumped into the tank = 718.67 – 50 = 668.67 kilolitres

Q12. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceeds its height ?

[NCERT Exemplar]

Sol. Let the radius of sphere and cylinder be r and h be the height of cylinder. Then according to the question.

Volume of sphere = Volume of cylinder

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi r^2 h \qquad \Rightarrow \quad r = \frac{3}{4}h$

Diameter of the cylinder $= \frac{3}{2}h$

Difference between the diameter and height of the cylinder $= \frac{3}{2}h - h = \frac{h}{2}$

Percentage by which the diameter exceeds the height of cylinder

$$= \frac{\frac{h}{2}}{4} x 100 = \frac{h}{2} x \frac{1}{h} x 100 = 50\%$$

Thus, the diameter of the cylinder exceeds its height by 50%

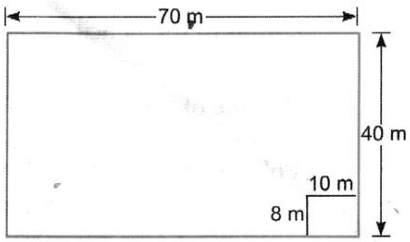
Q13. A field is 70 m long and 40 m broad. In the corner of the field, a pit which is 10 m long, 8 m broad and 5 m deep, has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Sol. Area of the field on which earth taken out is to be spread

$= 70 \text{ x } 40 \text{ m}^2 - 10 \text{ x } 8 \text{ m}^2$

$= 2800 \text{ m}^2 - 80 \text{ m}^2 = 2720 \text{ m}^2$

Volume of the earth dug out $= 10 \text{ x } 8 \text{ x } 5 \text{ m}^3 = 400 \text{ m}^3$



Rise in level of the field $= \frac{\text{Volume of the earth dugut}}{\text{A reaon which earth taken out is to}}$

$$= \frac{400}{2720} = 0.147\text{m} = 14.7\text{cm}$$

Q14. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 cm.

i) How many students can sit in the tent if a student on an average, occupies $\frac{5}{7}m^2$ on the ground ?

ii) Find the Volume of the cone. **[NCERT Exemplar]**

Sol. Let l m be the height of the conical tent.

Radius of the base of conical tent (r) = 5 m

i) Area of the circular base of the cone = $\lambda r^2 = \frac{22}{7} x 5^2 \text{ m}^2$

$$\text{Number of students} = \frac{\text{Area of the base}}{\text{Area occupied by one student}}$$

$$= \frac{\frac{22}{7} x 5 x 5 \text{ m}^2}{\frac{5}{7}\text{m}^2} = \frac{22}{7} x 5 x 5 x \frac{7}{5} = 110$$

ii) Also, curved surface area of cone = πrl

$\Rightarrow \quad 165 = \frac{22}{7} x 5 x l$

$\Rightarrow \quad l = \frac{165 x 7}{22 x 5} \qquad \Rightarrow \quad l = \frac{21}{2}\text{m} = 10.5 \text{ m}$

Also, $h^2 = l^2 - r^2$

$\Rightarrow \quad h = \sqrt{(10.5)^2 - 5^2} = \sqrt{15.5 x 5.5} = \sqrt{85.25} = 9.23\text{cm}$

Volume of conical tent = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \text{x} \frac{22}{7} \text{x} 5^2 \text{x} 9.23 \text{m}^3 = 241.74 \text{m}^3$$

Q15. The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Sol. Let the original diameter of the sphere be 2x.

Then, original radius of the sphere = x

Original curved surface area = $4\pi r^2$

Decreased diameter of the sphere = $= 2x - 25\% \text{ of } 2x = 2x - \frac{x}{2} = \frac{3}{2}x$

Decreased radius of the sphere $= \frac{3}{4}x$

$\therefore$ Decreased curved surface area $= 4\pi\left(\frac{3}{4}x\right)^2 = \frac{9}{4}\pi x^2$

Decreased in area $= 4\pi x^2 - \frac{9}{4}\pi x^2 = \frac{7}{4}\pi x^2$

Hence, percentage decrease in area = $\frac{\frac{7}{4}\pi x^2}{4\pi x^2} \text{x} 100\% = \frac{7}{16} \text{x} 100\% = \frac{175}{4}\% = 43.75\%$

Arithmetic progressions

1. **The fourth term of an AP is 0. Prove that its 25^{th} term is triple its 11^{th} term.**

   Ans: $a_4 = 0$

   $\Rightarrow$ a + 3d = 0

   T.P a25= 3 (a11)

   $\Rightarrow$ a + 24d = 3 (a + 10d)

   $\Rightarrow$ a + 24d = 3a + 30d

   RHS sub a = - 3d

   - 3d + 24d = 21d

   LHS 3a + 30d

   - 9d + 30d = 21d

   LHS = RHS. Hence proved

2. **Find the 20^{th} term from the end of the AP 3, 8, 13 …… 253.**

   Ans: 3, 8, 13 253

   Last term = 253

   a20 from end

   = l – (n-1)d

   253 – (20-1) 5

   253 – 95

   = 158

3. **If the p^{th}, q^{th} & r^{th} term of an AP is x, y and z respectively, show that x(q-r) + y(r-p) + z(p-q) = 0**

   Ans: p^{th} term $\Rightarrow$ x = A + (p-1) D

q^{th} term $\Rightarrow$ y = A + (q-1) D

r^{th} term $\Rightarrow$ z = A + (r-1) D

T.P x(q-r) + y(r-p) + z(p-q) = 0

={A+(p-1)D}(q-r) + {A + (q-1)D} (r-p)+ {A+(r-1)D} (p-q)A {(q-r) + (r-p) + (p-q)} + D {(p-1)(q-r)+ (r-1) (r-p) + (r-1) (p-q)}

⇒ A.0 + D{p(q-r) + q(r-p) + r (p-q)- (q-r) – (r-p)-(p-q)}

= A.0 + D.0 = 0. Hence proved

4. **Find the sum of first 40 positive integers divisible by 6 also find the sum of first 20 positive integers divisible by 5 or 6.**

   Ans: No's which are divisible by 6 are 6, 12.....240.

   S40 = [240] 6+240

   = 20 x 246

   = 4920

   No's div by 5 or 6

   30, 60 600

   [220] 30+600

   = 10 x 630

   = 6300

5. **A man arranges to pay a debt of Rs.3600 in 40 monthly instalments which are in a AP. When 30 instalments are paid he dies leaving one third of the debt unpaid. Find the value of the first instalment.**

   Ans: Let the value of I instalment be x S_{40} = 3600.

   $\Rightarrow \frac{40}{2}[2a + 39d] = 3600$

   ⇒2a + 39d = 180.....1

   $S_{30} = \frac{30}{2}[2a + 29d] = 2400$

   ⇒ 30a + 435d = 2400

   ⇒ 2a + 29d = 160 2

   Solve 1 & 2 to get

   d = 2 a = 51.

   ∴ I instalment = Rs.51.

6. **Find the sum of all 3 digit numbers which leave remainder 3 when divided by 5.**

   Ans: 103, 108 998

a + (n-1)d = 998

$\Rightarrow$ 103 + (n-1)5 = 998

$\Rightarrow$ n = 180

$S_{180} = \frac{180}{2}[103 + 998]$

S_{180} = 99090

7. **Find the sum of $(1 - \frac{1}{n}) + (1 - \frac{2}{n}) + (1 - \frac{3}{n})$ ..... upto n terms.**

Ans: $(1 - \frac{1}{n}) \quad (1 - \frac{2}{n}) +$ - upto n terms

$\Rightarrow$ [1+1+.....+n terms] $-[\frac{1}{n} + \frac{2}{n} +$ ....+ n terms]

n – [S_n up to n terms]

$S_n = \frac{n}{2}[2a + (n - 1d]$ $(d = \frac{1}{n}, a = \frac{1}{n})$

$= \frac{n}{2}\left[\frac{2}{n} + (n-1)\frac{1}{n}\right]$

$= \frac{n+1}{2}$ (on simplifying)

$n - \frac{n+1}{2}$

$= \frac{n-1}{2}$ Ans

8. **If the following terms form a AP. Find the common difference & write the next 3 terms3, 3+ √2, 3+2√2, 3+3√2.....**

Ans: d= $\sqrt{2}$ next three terms $3 + 4_{\backslash}/^2, 3 + 5_{\backslash}/^2, 3 + 6_{\backslash}/^2 \ldots\ldots$

9. **Find the sum of a+b, a-b, a-3b, ... to 22 terms.**

Ans: a + b, a – b, a – 3b, up to 22 terms

d = a – b – a – b = 2b

$S_{22} = \frac{22}{2}[2(a + b) + 21(-2b)]$

11[2a + 2b – 42b]

= 22a – 440b Ans.

10. **Write the next two terms √12, √27, √48, √75.....**

Ans: next two terms $\sqrt{108}, \sqrt{147}$ AP is $2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}\ldots\ldots$

11. **If the pth term of an AP is q and the qth term is p. P.T its nth term is (p+q-n).**

Ans: APQ

a_p = q

a_q = p

a_n =?

a + (p-1) d = q

a + (q-1) d = p

d [p – q] = q – p Sub d = -1 to get $\Rightarrow$ = -1 $\Rightarrow$ a = q + p -1

a_n = a + (n – 1)d

= a + (n - 1)d

= (q + p – 1) + (n – 1) - 1

a_n = (q + p – n)