

DELHI PUBLIC SCHOOL JAMMU
(SESSION 2019-20)
Pre- Board- I ASSIGNMENT

SUB: MATHEMATICS

CLASS: XII

- Q01. Show that $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $\begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$ is both one-one and onto.
- Q02. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1}$, for all $x \in \mathbb{R}$, is neither one-one nor onto.
- Q03. Let R be the relation defined on set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.
- Q04. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 10x + 7$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$ such that $g \circ f = f \circ g = I_{\mathbb{R}}$.
- Q05. Let A be the set of all students of class XII in a school and R be the relation, having the same sex on A , and then prove that R is an equivalence relation.
- Q06. If $y = \sin^{-1}(6x\sqrt{1-9x^2})$, then find $\frac{dy}{dx}$.
- Q07. Differentiate w.r.t x : $\cos^{-1} \left[\frac{3x+4\sqrt{1-x^2}}{5} \right]$.
- Q08. Find : $\int \frac{\cos \theta}{(4+\sin^2 \theta)(5-4\cos^2 \theta)} d\theta$.
- Q09. Evaluate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$.
- Q10. Evaluate: $\int_0^1 \frac{1}{e^x+e^{-x}} dx$.
- Q11. Evaluate: $\int_{-1}^{\frac{3}{2}} |x \sin(\pi x)| dx$.
- Q12. Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$
- Q13. Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$.
- Q14. Evaluate: $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$.
- Q15. Evaluate: $\int \frac{\log x}{x^2} dx$.
- Q16. Evaluate: $\int \frac{\sin x}{(\cos^2 x + 1)(\cos^2 x + 4)} dx$.
- Q17. If $\int_0^a 3x^2 dx = 8$, find the value of a .
- Q18. Evaluate: $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$
- Q19. Evaluate: $\int_0^{\frac{\pi}{2}} \sin 2x \log(\tan x) dx$.
- Q20. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$.
- Q21. Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

- Q22. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units . Show that the maximum volume of the box is $c^3/6\sqrt{3}$ cubic units .
- Q23. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.
- Q24. Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.
- Q25. A window of perimeter (including the base of the arc) is in the form of a rectangle surrounded by a semi circle. The semi-circular portion is fitted with coloured glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. Show that the ratio of the length and breadth of the rectangle is $6:6+\pi$, so that the window transmits maximum light.
- Q26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.
- Q27. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
- Q28. A jet of enemy is flying along the curve $y=x^2+2$ and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet.
- Q29. Find the area of the region $\{(x,y): 0 \leq y \leq x^2+1; 0 \leq y \leq x+1; 0 \leq x \leq 2\}$.
- Q30. Using integration, find the area of the region bounded by curves:
 $y=\sqrt{5-x^2}$ and $y=|x-1|$.
- Q31. Find the area of the region $\{(x,y): 0 \leq y \leq x^2+1; 0 \leq y \leq x+1; 0 \leq x \leq 2\}$
- Q32. Find the area enclosed between the parabola $4y=3x^2$ and the straight line $3x-2y+12=0$.
- Q33. Find the general solution of the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$.
- Q34. Find the general solution of the differential equation $(\cos x)y_1 - \cos 2x = \cos 3x$.
- Q35. Find the particular solution of the differential equation:
 $\frac{dx}{dy} + x \cot y = 2y + y^2 \cot y, (y \neq 0)$ given that $y(0) = \frac{\pi}{2}$.
- Q36. Solve the differential equation: $\tan y \frac{dy}{dx} = \cos(x+y) + \cos(x-y)$.
- Q37. If $y(x)$ is a solution of $\left(\frac{2+\sin x}{1+y}\right) \frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of y at $x = \frac{\pi}{2}$.
- Q38. Find the particular solution of the differential equation
 $\tan x \frac{dy}{dx} = 2x \tan x + x^2 - y; (\tan x \neq 0)$, given that $y=0$ when $x = \frac{\pi}{2}$.
- Q39. Solve: $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.
- Q40. Show that the differential equation $(x-y) dy = (x+2y) dx$ is homogenous. Also, find the general solution of the given differential equation.
- Q41. Find the particular solution of the differential equation $ye^y dx = (y^3 + 2xe^y) dy, y(0)=1$.
- Q42. Verify that $ax^2+by^2=1$ is a solution of the differential equation $x(yy_2+y_1^2) = yy_1$.

Q43. Find the differential equation representing the family of curves $y = ae^{bx+5}$, where a and b are arbitrary constants.

Q44. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} - y + \left(\frac{dy}{dx} + \frac{d^3y}{dx^3}\right)^{\frac{3}{2}}$.

Q45. If $x \frac{dy}{dx} = y(\log y - \log x)$, then find the solution of the equation.

Q46. Find the value of λ , if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are coplanar.

Q47. Two bikers are running at the Speed more than allowed speed on the road along the Lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$. Using Shortest distance formula check whether they meet to an accident or not?

Q48. If points (1,1,p) and (-3,0,1) be equidistant from the planes $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, then find the value of p.

Q49. Find the equation of the perpendicular drawn from the point (1, -2, 3) to the planes $2x - 3y + 4z + 9 = 0$. Also find the coordinates of the foot of perpendicular.

Q50. Find distance of the point (2, 3, 4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$, measured parallel to the plane $3x + 2y + 2z + 5 = 0$.

Q51. Find the equation of the plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$.

Q52. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of rs. 8000 on each piece of model A and 12000 on each piece of model B. Using LPP, solve the problem for maximum profit.

Q53. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel.

Q54. An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but there are 45000 bottles into which either of the medicine can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of

medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B and there are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan his production to cover maximum patients? How can you help others in case of natural disaster?

Q55. Vikas has been given two lists of problems from his mathematics teacher with the instructions to submit not more than 100 of them correctly solved for marks. The problems in the first list are worth 10 marks each and those in the second list are worth 5 marks each. Vikas knows from past experience that he requires on an average of 4 minutes to solve a problem of 10 marks and 2 minutes to solve a problem of 5 marks. He has other subjects to worry about; he cannot devote more than 4 hours to his mathematics assignment. With reference to manage his time in best possible way how many problems from each list shall he do to maximize his marks? What is the importance of time management for students?

Q56. A company manufactures two types of sweaters, type A and type B. It costs Rs. 360 to make one unit of type A and Rs. 120 to make a unit of type B. The company can make at most 300 sweaters and can spend at most Rs. 72000 a day. The number of sweaters of type A cannot exceed the number of type B by more than 100. The company makes a profit of Rs. 200 on each unit of type A but considering the difficulties of a common man the company charges a nominal profit of Rs. 20 on a unit of type B. Using LPP, solve the problem for maximum profit.

Q57. A manufacturing company makes two types of teaching aids A and B of Mathematics of Class XII. Each type of A requires 9 hrs for fabricating and 1 labour hours for finishing. Each type of B requires 12 hrs for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 80 on each piece of type A and Rs 120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to realize a maximum profit? What is the maximum profit per week?

Q58. An aero plane can carry a maximum of 200 passengers. A profit of Rs.1000 is made on each executive class ticket and a profit of Rs.600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. Form an LPP and solve it graphically.

Q59. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hostellers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hosteller?

Q60. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of number of aces. Also find mean of the distribution.

Q61. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then two balls are drawn at random (without replacement) from Bag II. The balls so drawn are found to be both red in colour. Find the probability that the transferred ball is red.

- Q62. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is strike. Determine the probability that the construction job will be completed on time.
- Q63. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin tossed twice find the probability distribution and mean of numbers of tails. Is this a good tendency? Justify your answer.
- Q64. A man is known to speak truth 5 out of 6 times. He draws a ball from the bag containing 4 white and 6 black balls and reports that it is white. Find the probability that it is actually white?
- Q65. A drunkard man takes a step forward with probability 0.6 and takes a step backward with Probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point.
- Q66. If group A contains the students who try to solve the problem by knowledge, Group B contains the students who guess to solve the problem Group C contains the students who give answer by cheating. If $n(A) = 20$, $n(B) = 15$, $n(C) = 10$, 2 Students are selected at random. Find the probability that they are from group c.
- Q67. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance?
- Q68. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six.
- Q69. There are 20 People in a group. Out of them 7 people are non-vegetarian, 2 people are selected randomly. Write the probability distribution of non-vegetarian people. Explain whether you would like to be vegetarian or non-vegetarian and why? Q12) Two third of the students in a class are sincere about their study and rest are careless Probability of passing in examination is 0.7 and 0.2 for sincere and careless students respectively, A Student is chosen and is found to be passed what is the probability that he/she was sincere.
- Q70. A company has two plants of scooter manufacturing. Plant I manufacture 70% Scooter and plant II manufactures 30%. At plant I 80% of the scooter's are maintaining pollution norms and in plant II 90% of the scooter maintaining Pollution norms. A Scooter is chosen at random and is found to be fit on pollution norms. What is the probability that it has come from plant II.
- Q71. A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job. Find the probability distribution for selection of persons other than their relatives. If the chairman is biased than which value of life will be demolished?
- Q72. A manufacturer has three machine operators A (skilled) B (Semi- skilled) and C (non skilled) The first operator A Produces 1% defective items where as the other two operators Band C produces 5% and 7 % defective items respectively. A is on the job for 50% of time B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced what is the probability that it was produced by B?
- Q73. In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families feel both children are equal.

Q74. In a group of 200 people, 50% believe in that anger and violence will ruin the country, 30% do not believe in that anger and violence will ruin the country and 20% are not sure about anything. If 3 people are selected at random find the probability that 2 people believe and 1 does not believe that anger and violence will ruin the country.

Q75. In a group of students, 200 attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers. The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with help of peers are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he study himself with help of peers.

Q76. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

Q77. A shopkeeper sells three types of flower seeds A1, A2 and A3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability of a randomly chosen seed to germinate.

Q78. A random variable X has the following probability distribution find (i) k (ii) $P(X \leq 1)$ (iii) $P(X > 3)$

X	0	1	2	3	4	5
P(X)	0.1	k	0.2	2k	0.3	k

Q79. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university: (i) none will graduate, (ii) only one will graduate and (iii) all will graduate.

Q80. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being a club?

Q81. There are 20 students out of which 8 come to school by their own cycles and rest by car. Two persons are selected at random out of them, write the probability distribution for the selected persons who are cyclists. Also find the mean of distribution.

Q82. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits the target is $\frac{2}{5}$. If each one of

A and B shoots at the target, what is the probability that (i) the target is hit? (ii) exactly one of them hits the target?

Q83. The probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

- (i) problem is solved
- (ii) exactly one of them solves the problem.

Q84. In an examination an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The prob. that he makes the guess is $(\frac{1}{3})$ and the probability that he copies the answer is $(\frac{1}{6})$. The probability that his answer is correct given that he copied it is $(\frac{1}{8})$. Find the prob. that he knew the answer to the question given that he correctly answered it.

Q85. A factory, has three types of machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, machine Y produces 1.5% and machine Z produces 2% defective bolts. At the end of a day, a bolt is picked at random and is found to be defective. Find the probability that this defective bolt is produced by the machine X.

Q86. Bag X contains 2 white and 3 red balls. Bag Y contains 5 white and 4 red balls. Bag Z contains 2 white and 3 red balls. A ball is drawn at random from one of the bags and it is found to be red. What is the probability that it is drawn from bag Y?

- Q87. A lab blood test is 99% effective in detecting a certain disease when it is in fact present. However, it also yields a false positive result for 0.5% of the healthy person tested. If 0.1% of the population actually has the disease, what is the probability that a person has the disease, given that his test result is positive?
- Q88. Three persons A, B and C fire a target in turn, starting with A. Their probability of hitting the target is 0.5, 0.3 and 0.2 respectively. Find the probability of at most one hit. In life we must set a target/Goal. To achieve the target we need to follow some values/qualities. Mention any two such qualities.
- Q89. A and B throw alternatively with a pair of dice. A wins if he throws a sum 6 before B and B wins if he throws a sum 7 before A throws a sum 6. If A begins, Show that the odds in favor of A are 30:31.
- Q90. Suppose you have two coins which appear identical in your pocket. You know that one is fair and the other is two-headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?