Delhi Public School, Jammu
Question Bank
(2017 – 18)

Class : XI **Subject : Maths**

1. Find the number of subsets of a set A containing 10 elements

Solution

Number of subsets

$^{10}C_0 +^{10} C_1 +^{10} C_2 +^{10} C_3 +^{10} C_4 +^{10} C_5 +^{10} C_6 +^{10} C_7 +^{10} C_8 +^{10} C_9 +^{10} C_{10} = 2^{10}$

2. How many ways can you choose one or more students from 3 students?

Solution

$^3C_1 +^3 C_2 +^3 C_3 + = 2^3 - 1 \ = 7$

3. In How many ways can one choose 3 cards from a pack of 52 cards in succession (1) with replacement (2) without replacement?

Solution

1. Each card can be drawn in 52 ways and so the total number of ways =

   $52 \times 52 \times 52 = 52^3$

2. If there is no replacement the first card can be drawn in 52 ways, the second by 51 ways and the third by 50 ways. Hence the total number of ways is

   $52 \times 51 \times 50 = 132600$

4. State the condition under which the product of two complex numbers is purely imaginary.

Solution:

1. None of the factors are zero
2. Factors must be of the form $(a+ib); k(b+ia)$ where k is a real number

5. In a circle of radius 1 unit what is the length of the arc that submits an angle of 2 radians at the centre.

Solution

Length of arc = $r\theta$

Hence length of arc==2units

6. Is $\cos\theta$ positive or negative if $\theta = 500$ radians

Solution

1 Full rotation is $2\pi radians$

500 radians = $\dfrac{500}{2\pi}$ rotations $= 79.57 rotations$ Or 79 full rotations and 0.57 of a rotation

$0.5 < 0.57 < 0.75$

The incomplete rotation is between $\dfrac{1}{2} and \dfrac{3}{4}$ of a rotation. Hence 500 radians is in third quadrant. So $\cos\theta$ is negative

7. Solve $\sin^2 x + \sin^2 2x = 1$

Solution:

$$\frac{1-\cos 2x}{2} + \frac{1-\cos 4x}{2} = 1$$

cos2x + cos4x = 0

2 cos3x cos x = 0

Cos3x = 0

$$x = \frac{\pi}{6} + \frac{\pi}{3}n$$

$$Cos x = 0$$

$$x = \frac{\pi}{2} + \pi k = \frac{\pi}{6} + \frac{\pi}{3}n \quad n \text{ is integer}$$

8. Find the value of $i^{30} + i^{40} + i^{60}$

Solution:

$$i^{30} + i^{40} + i^{60} = (i^4)^7 . i^2 + (i^4)^{10} + (i^4)^{15}$$

$$i^4 = 1 = -1 + 1 + 1 = 1$$

9. Determine whether the points $(0,0)$ and $(5,5)$ lie on different sides of the straight line $x + y - 8 = 0$ or on the same side of the straight line.

Solution:

Substituting the points (0, 0) and (5, 5) on the given line

x + y – 8 = 0

0 + 0 – 8 = -8

5 + 5 – 8 = 2

Since the signs of the resulting numbers are different the given points lie on opposite sides of the given line.

10. Prove that $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

Solution :

$\tan^{-1} x = A$

$\tan A = x$

$\cot^{-1} x = B$

$\cot B = x$

$\tan(\frac{\pi}{2} - B) = x$

$\tan^{-1} x = \frac{\pi}{2} - B$

$\tan^{-1} x = A$

$A = \frac{\pi}{2} - B$

$A + B = \frac{\pi}{2}$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

11. Prove by mathematical induction that $11^{n+2} + 12^{2n+1}$ is divisible by 133 for all positive integer values of n

Solution:

$11^{n+2}+12^{2n+1}$ is divisible by 133

n = 1

$11^3+12^3=(11+12)(11^2-11.12+12^2)$

=23.133

Let it be true for k

$11^{k+2}+12^{2k+1}$ is divisible by 133

For k = k + 1

$11^{k+3}+12^{2k+3}=11.11^{k+2}+12^2.12^{2k+1}$

$=11.11^{k+2}+144.12^{2k+1}$

$=11.11^{k+2}+133.12^{2k+1}+11.12^{2k+1}$

$=11.11^{k+2}+11.12^{2k+1}+133.12^{2k+1}$

Is divisible by 133 since $11^{k+2}+12^{2k+1}$ is divisible by 133

12. A, B, C are 3 sets and U is the universal set such that

$n(U)=800, n(A)=200, n(B)=300, n(A\cap B)=100$ Find $n(A'\cap B')$

Solution:

$n(A'\cap B')=n(A\cup B)'=n(U)-n(A\cup B)$

$=n(U)-[n(A)+n(B)-n(A\cap B)]$

$800-[200+300-100]$

$=400$

13. If α,β are the roots of the equation $x^2-bx+c=0$ find the value of $\alpha^2+\beta^2$

Solution: $\alpha+\beta=b$

$\alpha\beta=c$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=b^2-2c$

14. If P be the sum of the odd terms and Q the sum of the even terms in the expansion of $(x+a)^n$,

prove that $P^2-Q^2=(x^2-a^2)^n$

Solution:

$(x + a)^n = P + Q$

$$(x-a)^n = P - Q$$
$$(P+Q)(P-Q) = (x+a)(x-a)$$
$$P^2 - Q^2 = (x^2-a^2)^n$$

15. Solve the inequality $\frac{x^2-3x+6}{3+4x} < 0$

Solution:

Discriminant of numerator = 9 – 24 < and

Coefficient of x^2 is positive. Hence Numerator is always positive

Hence dividing by the numerator on both sides of

The equality does not change the sign of the inequality

Hence we need only consider $\frac{1}{3x+4} < 0$

$$x < \frac{-4}{3}$$
$$x \in (-\infty, -\frac{4}{3})$$

16. Prove that $\cot(A+15) - \tan(A-15) = \frac{4\cos 2A}{1+2\sin 2A}$

Solution:

$$\cot(A+15) - \tan(A-15) = \frac{\cos(A+15)}{\sin(A+15)} - \frac{\sin(A-15)}{\cos(A-15)}$$
$$= \frac{\cos(A+15)\cos(A-15) - \sin(A+15)\sin(A-15)}{\sin(A+15)\cos(A-15)}$$
$$= \frac{\cos 2A}{\frac{1}{2}(\sin 2A + \frac{1}{2})}$$
$$= \frac{2\cos 2A}{\sin 2A + \frac{1}{2}}$$
$$= \frac{4\cos 2A}{1+2\sin 2A}$$

17. Find the domain of the function $f(x) = \sqrt{4-x^2}$

Solution:

$$4-x^2 \geq 0$$
$$x^2-4 \leq 0$$
$$Domain\ of\ \ x \in [-2,2]$$

$$y^2 = 4-x^2$$
$$x^2 = 4-y^2$$
$$x = \sqrt{4-y^2}$$
$$4-y^2 \geq 0$$
$$y^2-4 \leq 0$$
$$y \in [-2,2]$$

Also for all values of $x \in [-2,2]$

$$y = \sqrt{4-x^2} \geq 0$$
$$Range\ \ y \in [0,2]$$

18. Evaluate $\frac{1}{2+\cos\theta+\sin\theta}$ if $\tan\frac{\theta}{2} = 2$

Solution:

$$\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$
$$\cos\theta = \frac{1-4}{1+4} = \frac{-3}{5}$$
$$\sin\theta = \frac{2\tan\theta 2}{1+\tan^2\frac{\theta}{2}} = \frac{2.2}{1+4} = \frac{4}{5}$$

$$\frac{1}{2+\cos\theta+\sin\theta} = \frac{1}{2-\frac{3}{5}+\frac{4}{5}} = \frac{11}{5}$$

19. Find the limit $\lim_{x\to 0}\frac{\sin 5x}{x+x^3}$

Solution:

$$\lim_{x\to 0}\frac{\sin 5x}{x+x^3}=\lim_{x\to 0}\frac{5\sin 5x}{5x(1+x^2)}$$

$$=\lim_{x\to 0}\frac{5\sin 5x}{5x}\lim_{x\to 0}\frac{1}{(1+x^2)}$$

$$=5.1.1$$

$$=5$$

20. Differentiate $\log_{10} x$ with respect to x

Solution :

$$y=\log_{10} x$$

$$x=10^y$$

$$\log_e x = y\log_e 10$$

$$y=\frac{\log_e x}{\log_e 10}$$

$$\frac{dy}{dx}=\left(\frac{1}{\log_e 10}\right)\frac{1}{x}$$

21. How many 6 digits numbers can be formed with the digits $1, 2, 3, 4, 5, 6, 7$ if the 10th, unit's places are always even and repetition is not allowed.

Solution:

There are 3 even numbers 2, 4, 6

So the units place, 10th places can be filled in $3p_2$ ways

Remaining 5 digits can be used to fill 4 places in $5p_4$ ways.

Hence the total numbers satisfying the above condition is $3p_2\times 5p_4=720$

22. Shift the origin to a suitable point so that the equation $x^2+y^2-4x+6y=36$ representing a circle is transformed in to an equation of a circle with centre at origin in the new coordinate axes.

Let the origin be shifted to (h, k)

$$x = x' + h$$
$$y = y' + k$$

Then

$$(x'+h)^2 + (y'+k)^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + 2hx' + h^2 + y'^2 + 2ky' + k^2 - 4(x'+h) + 6(y'+k) = 36$$
$$x'^2 + y'^2 + x'(2h-4) + y'(2k+6) + h^2 + k^2 - 4h + 6k - 36 = 0$$

2h – 4 = 0

h = 2

2k + 6 = 0

k = -3

$$x'^2 + y'^2 + 2^2 + (-3)^2 - 8 - 18 - 36 = 0$$
$$x'^2 + y'^2 + 13 - 62 = 0$$
$$x'^2 + y'^2 = 49$$

23. The mean and variance of 7 observations are 8 and 19 respectively. If 5 of the observations are 2, 4,12,14,11. Find the remaining observations.

Solution:

$$\frac{2+4+12+14+11+x+y}{7} = 8$$
$$43 + x + y = 56$$
$$x + y = 13$$

$$\frac{2^2+4^2+12^2+14^2+11^2+x^2+y^2}{7} - (mean)^2 = 19$$
$$\frac{4+16+144+196+121+x^2+y^2}{7} - 64 = 19$$
$$\frac{481+x^2+y^2}{7} = 83$$

$$481 + x^2 + y^2 = 581$$
$$x^2 + y^2 = 100$$
$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$169 + (x-y)^2 = 200$$
$$(x-y)^2 = 31$$
$$x - y = 5.57$$

$x+y=13$
$x=9.285$
$y=3.715$

24. Prove that $\frac{1}{\log_a b}, \frac{1}{\log_{2a} b}, \frac{1}{\log_{4a} b}$ form an AP

Solution:

$$\frac{1}{\log_a b}=\log_b a$$
$$\frac{1}{\log_{2a} b}=\log_b 2a$$
$$\frac{1}{\log_{4a} b}=\log_b 4a$$
$$\frac{\log_b a+\log_b 4a}{2}=\frac{\log_b (2a)^2}{2}$$
$$=2\frac{\log_b 2a}{2}$$
$$=\log_b 2a$$

Thus, $\frac{1}{\log_{2a} b}$ is, the, AM, between $\frac{1}{\log_a b}, \frac{1}{\log_{4a} b}$

25. On the average one person dies out of every 10 accidents find the probability that at least 4 will be safe out of 5 accidents.

Solution:

Probability of surviving = $\frac{9}{10}$

Required to find out the probability of 4 are safe or 5 are safe

Probability of 5 is safe = $\left(\frac{9}{10}\right)^5$

Probability of 4 is safe = ${}^5C_4\left(\frac{9}{10}\right)^4\frac{1}{10}$

Required Probability = $\left(\frac{9}{10}\right)^5+5\left(\frac{9}{10}\right)^4\frac{1}{10}=\frac{45927}{5000}$