

CLASS 12TH

SECTION A (1 mark each)

Q1. If A is a square matrix of order 3 and $|adj A| = 144$, then find the value of $|3A|$.

Q2. If \vec{p} is a unit vector and $(\vec{x} - \vec{p}) \cdot (\vec{x} + \vec{p}) = 80$, then find the value of $|\vec{x}|$.

Q3. Evaluate $\int \frac{\sec^2(\log x)}{x} dx$.

Q4. Differentiate $\log(x + \sqrt{x^2 + a^2})$ w.r.t to x.

SECTION B (2 marks each)

Q5. Using elementary transformation find the inverse of matrix $A = \begin{pmatrix} 5 & 4 \\ 3 & 2 \end{pmatrix}$

Q6. Using differentials find the approximate value of $f(5.0001)$

Where $f(x) = x^3 - 7x^2 + 15$

Q7. Find the equation of tangents to the curve $3x^2 - y^2 = 8$. Which passes through the point $(4/3, 0)$

Q8. If $\cos y = x \cos(a+y)$ prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

Q9. Find the coordinate of point of intersection $\frac{x+1}{2} = \frac{y+3}{3} = \frac{z+3}{4}$ meets the plane $x+y+4z = 6$.

Q10. Let A and B are two events such that $P(\overline{A \cap B}) = \frac{1}{6} P(A \cap B) = \frac{1}{4}$. Prove that A and B are independent events.

Q11. Solve for x, $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

Q12. Evaluate $\int \frac{x-4}{(x-2)^3} e^x dx$.

SECTION C (4 marks each)

Q13. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b}\right) = \frac{2b}{a}$.

Q14. Using the properties of determinants, prove that:

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

Q15. If $x = a(\cos t + \log \tan \frac{t}{2})$, $y = a \sin t$ find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

Q16. Evaluate $\int \frac{x^2-1}{x^4+1} dx$

Q17. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ (or)

Q18. Form the differential equation of the family of curves in the first quadrant which touches the coordinate axes.

Q19. Given $\vec{a} = 3\hat{i} - \hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{b} = \vec{b}_1 + \vec{b}_2$ where \vec{b}_1 is parallel to \vec{a} and \vec{b}_2 is perpendicular to \vec{a}

Q20. If \vec{a}, \vec{b} and \vec{c} are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 5$ then prove that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$.

Q21. An instructor has a question bank consisting of 300 easy true / false questions ,200 difficult true/false questions,500 easy multiple choice questions and 400 difficult multiple choice questions .If question is selected at random from the question bank,what is the probability that will be an easy question that it is a multiple choice question.

Q22.An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motorcycle is 0.02. An insured vehicle met with an accident.Find the probability that the accidented vehicle was a motorcycle .How we can avoid accidents ?

Q23. Let $F:N \rightarrow N$ such that : $F(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$.State whether the function F is bijective or not.

SECTION D (6 marks each)

Q24. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given,show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

Q25.Find the area of the region included between the parabola $y^2 = x$ and the line $x+y = 2$ using integration.

Q26.If $A = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 2 \\ 1 & -3 & 1 \end{bmatrix}$, find A^{-1} . Hence solve the system of linear equations $3x-2y+z = 2$, $2x+y-3z = -5$,
 $-x+2y+z = 6$.

Q27. A company manufacturers has two types of toys.Toys of type A requires 5 minutes each for cutting and 10 minutes each for assembling.Toys of type B requires each for cutting and 8 minutes each for assembling.There are 3 hours 20 minutes available for cutting and 4 hours available for assembling.The profit is rupees 0.50 each for type A and rupees 0.60 each for type B toys.How many toys of each type should be manufactured in order to maximize the profit?

Q28.The points $A(4,5,1)$ $B(2,3,4)$ and $C(1,2,-1)$ are three vertices of a parallelogram ABCD.Find the vector and Cartesian equation of the sides AB and BC and find coordinates of D .

Q29.Solve the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$