DELHI PUBLIC SCHOOL, JAMMU

ASSIGNMENT FOR PRE-BOARD -II

Sub: Mathematics **Class: XII**

Q1. Check whether the relation R in R of real numbers defined by R = {(a , b) : a $\leq b^3$} is reflexive, symmetric or transitive.

Q2. Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b} is
. an equivalence relation. Also find its all possible equivalence classes.

Q3. F : R → R, g : R → given by f(x) = [x], g(x) = $|x|$, then find (fog) $\left(\frac{-2}{3}\right)$ and (gof)$\left(\frac{-2}{3}\right)$.

Q4. Show that the function f : N → N defined by f(x) $\begin{cases} \frac{n+1}{2}, n is\ odd \\ \frac{n}{2}, n is\ even \end{cases}$ is not bijective.

Q5. Consider f: $R_+ \rightarrow [-5, \infty)$ given by f(x) = $9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

Q6. Prove that $\tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Q7. Prove that $\cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$.

Q8. If *A* and *B* are symmetric matrices, determine whether AB – BA is symmetric or skew – symmetric matrix.

Q9. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

Q10. Obtain the inverse of the matrix A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

Q11. There are three families A, B and C. The number of men, women and children in these families are as under:

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are Rs.200, Rs.150 and Rs.200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family

Q12. If A is a square matrix of order 3 x 3 such that $|A| = 5$, then find.$|4A|$

Q13. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that 1 + xyz = 0.

Q14. Prove that $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$.

Q15. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, then prove that a + b + c = 0 or a = b = c.

Q16. Solve the system of equations by using matrix method :

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

Q17. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school wants to award Rs.x each, Rs.z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs.1600. School B wants to spend Rs.2300 to award its 4, 1 and 3 students on the respective values(by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs.900, using matrices, find the award money for each value.

Q18. Show that the function *f* defined by f(x) = $\begin{cases} \frac{|x-4|}{x-4}, x \neq 4 \end{cases}$ is continuous every where except at x = 4.

Q19. Determine the value of *k* for which the function f(x) = $\begin{cases} \frac{1-\cos 2x}{2k^2}, x \neq 0 \\ k, x = 0 \end{cases}$ is continuous at x = 0.

Q20. Determine the values of a, b and c for which the function f(x) = $\begin{cases} \frac{\sin(a+1)x + \sin x}{x}, x < 0 \\ c, x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, x > 0 \end{cases}$ is continuous at x = 0.

Q21. Prove that the function *f* given by f(x) = $|x-1|, x \in R$ is continuous at x = 1 but not differentiable at x = 1.

Q22. Find $\frac{dy}{dx}, if\ y = (\sin x)^{\cot x} + (\sin x)^{sec x}$.

Q23. Find $\frac{dy}{dx}, if$ $x^y + y^x = a^b$.

Q24. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for -1 < x < 1, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

Q25. Verify Rolle"s Theorem for the function y = $x^2 + 2$ in the interval [-2, 2].

Q26. A particle moves along the curve y = $\frac{2}{3}x^3 + 1$. Find the point on the curve at which y – coordinate is changing twice as fast as x-coordinate.

Q27. Prove that the function f(x) = $\frac{x^3}{3} - x^2 + 9x, x \in [1,2]$ is strictly increasing. Hence find the minimum value of f(x).

Q28. Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve y = $be^{-\frac{x}{a}}$ at the point where the curve crosses the y-axis.

Q29. For the function f(x) = $-2x^3 - 9x^2 - 12x + 1$, find the intervals in which f(x) is i) increasing ii) decreasing.

Q30. Find the equation of the tangent and normal to the curve y = $\sin^2 x$ at the point $\left(\frac{\pi}{4}, \frac{3}{4}\right)$.

Q31. Evaluate: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.

Q32. Evaluate: $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - cos^2\theta - 4\,sin\theta} d\theta$.

Q33. Evaluate: $\int \frac{x+2}{2x^2+6x+5} dx$.

Q34. Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)} . dx$.

Q35 In a game, a man wins Rs. 5 for getting a number greater than 4 and loses Rs. 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

Q36. In answering a question on a multiple choice test, a student either knows the answer or guesses, Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer given that he answered it correctly?

Q37. Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\overset{\frown}{b} = \hat{\imath}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, then

a. Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

b. If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

Q38. Show that the points, A, B, C with position vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Q39. Find the vector equation of a line which is parallel to the vector $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and which passes through the point (5, - 2, 4). Also, reduce it to cartestan form.

Q40. A line pass through the point with position vector $2\hat{\imath} - 3\hat{J} + 4\hat{k}$ and is in the direction of $3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$. Find equation of the line in vector and cartesian form.

Q41. Find the equations of the line passing through the point (2, 1, 3) and perpendicular to the lines. $\frac{x-1}{1} = \frac{y-2}{2} = \frac{Z-3}{3} and \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

Q42. A fruit grower can use two types of fertilisers in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs atleast 240 kg of phosphoric acidm atleast 270 kg of potash and almost 310 kg of chlorine. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximim amount of nitrogen added?

	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Q43. Find the area of the region bounded by a circle $4x^2 + 4y^2 = 9$ and the parabola $y^2 = 4x$.

Q44. Solve the following initial value problem. $2xy + y^2 – 2x^2 \frac{dy}{dx} = 0, y(1) = 2$.

Q45. Solve the following differential equation $(1 + y + x^2y)dx + (x + x^3) dy = 0$