DELHI PUBLIC SCHOOL

JAMMU

Session 2018-19

QUESTION BANK

MATHEMATICS

CONTENTS

CHAPTER - 1 REAL NUMBERS

Q1. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start from the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

Sol. To find the time after which tye meet again at the starting point, we have to find LCM of 18 and 12 minutes. We have

2	18
3	9
3	3
	1

2	12
2	6
3	3
	1

$18 = 2 \times 3^2$

and $12 = 2^2 \times 3$

Therefore, LCM of 18 and 12 = $2^2 \times 3^2 = 36$

So, they will meet again at the starting point after 36 minutes.

Q2. If n is an odd positive integer, show that $(n^2 - 1)$ is divisible by 8.

Sol. We know that an odd positive integer n is of the form (4q + 1) or (4q + 3) for some integer q.

Case-I When n = (4q + 1)

In the case n2 – 1 = $(4q + 1)^2 - 1 = 16q^2 + 8q = 8q(2q + 1)$

Which is clearly divisible by 8.

Case - II When n = (4q + 3)

In this case, we have

$n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8(2q^2 + 3q + 1)$

which is clearly divisible by 8.

Hence $(n^2 - 1)$ is divisble by 8.

Q3. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

Sol. Let HCF of the number be x then according to question LCM of the number will be 14x

And $x + 14x = 600 \Rightarrow 15x = 600 \Rightarrow x = 40$

Then HCF = 40 and LCM = 14 x 40 = 560

$\therefore$ LCM x HCF = Product of the numbers

$$560 \times 40 = 280 \times \text{Second number} \Rightarrow \frac{560 \times 40}{280} = 80$$

Then other number is 80.

Q4. Show that $5-\sqrt{3}$ is an irrational number.

Sol. Let us assume that $5-\sqrt{3}$ is rational.

So, $5-\sqrt{3}$ may be written as

$5-\sqrt{3}=\frac{p}{q}$, where p and q are integers, having no common factor except 1 and q $\neq 0$

$\Rightarrow \quad 5-\frac{p}{q}=\sqrt{3} \qquad \Rightarrow \quad \sqrt{3}-\frac{5q-p}{q}$

Since $\frac{5q-p}{q}$ is a rational number as p and q are integers.

$\therefore$ $\sqrt{3}$ is also a rational number which is a contradiction.

Thus, our assumption is wrong.

Hence, $5-\sqrt{3}$ is an irrational number.

Q5. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.

Sol. It is given that on dividing 398 by the required number, there is a remainder of 7. This means that 398 – 7 = 391 is excactly divisible by the required number. In other words, required number is a factor of 391.

Similarly, required positive integer is factor of 436 – 11 = 425 and 542 – 15 = 527

Clearly, required number is the HCF of 391, 425 and 527.

Using the factor tree, we get the prime factorisations of 391, 425 and 527 as follows :

391 = 17 x 23, 425 = 5^2 x 17 and 527 = 17 x 31

$\therefore$ HCF of 391, 425 and 527 is 17.

Hence, required number = 17

CHAPTER - 2 : POLYNOMIALS

Q6. If the product of two zeroes of the polynomial p(x) = $2x^3 + 6x^2 - 4x + 9$ is 3,, then find its third zero.

Sol. Let α, β, γ be the roots of the given polynomial and $\alpha\beta = 3$

Then, $\alpha\beta\gamma = -\frac{9}{2}$

$\Rightarrow \quad 3 \text{ x } \gamma = \frac{-9}{2}$

or $\quad \gamma = \frac{-3}{2}$

Q7. If α and β are the zeros of the quadratic polynomials $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

Sol. Since α + β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$

$$\therefore \quad \alpha + \beta = \frac{-(-5)}{2} = \frac{5}{2} \text{ and } \alpha\beta = \frac{7}{2}$$

Let S and P denote respectively the sum and products of the zeros of the required polynomial.

Then, $S = (2\alpha + 3\beta) + (3\alpha + 2\beta) = 5(\alpha + \beta) = 5 \text{ x } \frac{5}{2} = \frac{25}{2}$

and $\quad P = (2\alpha + 3\beta)(3\alpha + 2\beta)$

$\Rightarrow \quad P = 6\alpha^2 + 6\beta^2 + 13\alpha\beta = 6\alpha^2 + 6\beta^2 + 12\alpha\beta + \alpha\beta$

$= 6(\alpha^2 + \beta^2 + 2\alpha\beta) + \alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$

$\Rightarrow \quad P = 6\text{x}\left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = 41$

Hence, the required polynomial g(x) is given by

$g(x) = k(x^2 - sx + P)$

or $\quad g(x) = k\left(x^2 - \frac{25}{2}x + 41\right)$, where k is any non-zero real number.

Q8. What must be subtracted from $p(x) = 8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisble by $g(x) = 4x^2 + 3x - 2$?

Sol. Let y be subtracted from polynomial p(x)

$\therefore$ $8x^4 + 14x^3 - 2x^2 + 7x - 8 - y$ is exactly divisble by g(x)

Now,
$$4x^2+3x-2\overline{)8x^4+14x^3-2x^2+7x-8-y}\ \ (2x^2+2x-1$$

$$\begin{array}{l} -8x^4 \pm 6x^3 \mp 4x^2 \\ \hline 8x^3+2x^2+7x-8-y \end{array}$$

$$\begin{array}{l} -8x^3 \pm 6x^2 \mp 4x \\ \hline -4x^2+11x-8-y \end{array}$$

$$\begin{array}{l} \pm 4x^2 \mp 3x \mp 2 \\ \hline 14x-10-y \end{array}$$

$\therefore$ Remainder should be 0.

14x - 10 - y = 0

or 14x - 10 = y or y = 14x - 10

$\therefore$ (14x - 10) should be subtracted from p(x) so that it will be exactly divisible by g(x)

Q9. Obtain the zeros of quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify the relation between its zeros and coefficients.

Sol. We have,

$$f(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$$

$$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$

$$= \sqrt{3}x\left(x - 2\sqrt{3}\right) - 2\left(x - 2\sqrt{3}\right)$$

$\Rightarrow$ $\left(x - 2\sqrt{3}\right)\left(\sqrt{3}x - 2\right) = 0$

$x = 2\sqrt{3}$ or $x = \dfrac{2}{\sqrt{3}}$

So, the zeros of $f(x)$ are $2\sqrt{3}$ and $\dfrac{2}{\sqrt{3}}$

Sum of zeros $2\sqrt{3} + \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}} = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

Product of the zeros $= 2\sqrt{3}\ x\ \frac{2}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = -\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence verified.

Q10. If one zero of polynomial (a^2 + 9) x^2 + 13x + 6a is reciprocal of the other, find the value of a.

Sol. Let one zero of the given polynomial be a.

Then, the other zero is $\frac{1}{\alpha}$

$\therefore$ Product of zeros = $\alpha \text{ x } \frac{1}{\alpha} = 1$

But, as per the given polynomial product of zeros $= \frac{6a}{a^2 + 9}$

$\therefore$ $= \frac{6a}{a^2+9} = 1$

$\Rightarrow$ $a^2 + 9 = 6a$

$\Rightarrow$ $a^2 - 6a + 9 = 0$ $\Rightarrow$ $(a - 3)^2 = 0$

$\Rightarrow$ $a - 3 = 0$ $\Rightarrow$ $a = 3$

Hence, a = 3.

CHAPTER - 3 : PAIR OF LINEAR EQUATION IN TWO VARIABLES

Q11. Solve for x and y

$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2; \quad x + y = 2ab$

Sol. We have, $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$ ...(i)

$x + y = 2ab$ ...(ii)

Multiplying (ii) by b/a, we gete

$$\frac{b}{a}x + \frac{a}{b}y = 2b^2 \qquad ...(iii)$$

Subtracting (iii) from (i), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right)y = a^2 + b^2 - 2b^2 \quad \Rightarrow \quad \left(\frac{a^2 - b^2}{ab}\right)y = \left(a^2 + b^2\right)$$

$$\Rightarrow \quad y = \left(a^2 + b^2\right) x \frac{ab}{\left(a^2 - b^2\right)} \quad \Rightarrow \quad y = ab$$

Putting the value of y in (ii), we get

x + ab = sab $\Rightarrow$ x = 2ab – ab

$\Rightarrow$ x = ab

$\therefore$ x = ab, y = ab

Q12. Five years ago, A was thrice as old as B and ten years later. A shall be twice as old as B. What are the present ages of A and B ?

Sol. Let the present ages of B and A be x years and y years respectively. Then

B's age 5 years ago = (x – 5) years

and A's age 5 years ago = (y – 5) years

(y – 5) = 3 (x – 5) $\Rightarrow$ 3x – y = 10 ... (i)

B's age 10 years hence = (x + 10) years

A's age 10 years hence = (y + 10) years

y + 10 = 2(x + 10) $\Rightarrow$ 2x – y = – 10 .... (ii)

On subtracting (ii) from (i) we get x = 20

Putting x = 20 in (i) we get

(3 x 20) – y = 10 $\Rightarrow$ y = 50

$\therefore$ x = 20 and y = 50

Hence, B's present age = 20 years and A's present age - 50 years.

Q13. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the numerator be x and denominator be y.

$\therefore$ Fraction = $\frac{x}{y}$

Now, according to question.

$\frac{x-1}{y} = \frac{1}{3}$ $\Rightarrow$ 3x – 3 = y

3x – y = 3 ---(i)

and $\frac{x}{y+8} = \frac{1}{4}$ $\Rightarrow$ 4x = y + 8 ---(ii)

yx – y = 8

Now, subtracting equation (iii) and (i), we have

$3x - y = 3$

4x – y = 8

– + –

– x = –5

$\therefore$ x = 5

Putting the value of x in equation (i), we have

3 x 5 - y = 3 15 – y = 3 15 – 3 = y

$\therefore$ y = 12

Hence, the required fraction is $\frac{5}{12}$

Q14. Show graphically the given system of equations

2x + 4y = 10 and 3x + 6y = 12

has no solution.

Sol. We have, 2x + 4y = 10

$\Rightarrow$ 4y = 10 – 2x $\Rightarrow$ $y = \frac{5-x}{2}$

Thus, we have the following table

x	1	3	5
7	2	1	0

Plot the point A (1, 2) B (3, 1) and C (5, 0) on the graph paper. Join A, B and C and extend it on both sides to obtain the graph of the equation 2x + 4y = 10

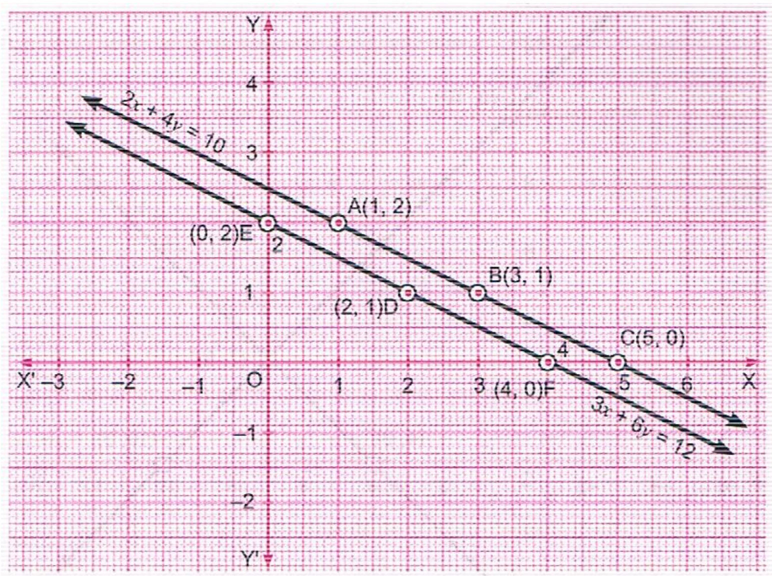
We have, $3x + 6y = 12$

$\Rightarrow$ $6y = 12 - 3x$ $\Rightarrow$ $y = \frac{4-x}{2}$

Thus, we have the following table

x	2	0	4
y	1	2	0

Plot the point D (2, 1), E (0, 2) and F (4, 0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation 3x + 6y = 12



We find that the lines represented by equation 2x + 4y = 10 and 3x + 6y = 12 are parallel. So, the two lines have no common point. Hence, the given system of equations has no solutions.

Q15. Solve the following pairs of linear equations by the elimination method and the substitution method :

$$\frac{x}{2} + \frac{2y}{3} = -1 \quad \text{and} \quad x - \frac{y}{3} = 3$$

Sol. We have, $\frac{x}{2} + \frac{2y}{3} = -1$ $\Rightarrow$ $\frac{3x + 4y}{6} = -1$

$\therefore \quad 3x + 4y = -6$ ---- (i)

and $\quad x - \frac{y}{3} = 3 \quad \Rightarrow \quad \frac{3x - y}{3} = 3$

$\therefore \quad 3x - y = 9$ ----(ii)

By Elimination Method :

Subtracting (ii) from (i), we have

$5y = -15$ or $y = -\frac{15}{5} = -3$

Putting the value of y in equation (i), we have

$3x + 4 \times (-3) = -6 \quad \Rightarrow \quad 3x - 12 = -6$

$\therefore \quad 3x = -6 + 12 \quad \Rightarrow \quad 3x = 6$

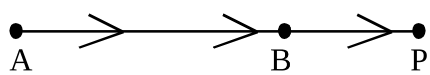
$\therefore \quad x = \frac{6}{3} = 2$

Hence, solution is x = 2, y = –3.

Q16. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cards.

Sol. Let the speed of two cars be x km/h and y km/h respectively.

Case I: When two cars move inthe same direction, they will meet each other at P after 5 hours.



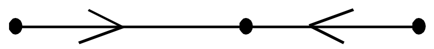
The distance covered by car from A = 5x (Distance = Speed x Time)

and distance covered by the car from B = 5y

$\therefore \quad 5x - 5y = AB = 100 \quad \Rightarrow \quad x - y = \frac{100}{5}$

$\therefore \quad x - y = 20$ --- (i)

Case II: When two cards move in opposite directions, they will meet each other at Q after one hour



The distance covered by the car from A = x

The distance covered by the car from B = y

$\therefore$ x + y = AB = 100 $\Rightarrow$ x + y = 100 ---(ii)

Now, adding equations (i) and (ii), we have

$$2x = 120 \quad \Rightarrow \quad x = \frac{120}{2} = 60$$

Putting the value of x in equation (i), we get

$$60 - y = 20 \quad \Rightarrow \quad -y = -40 \quad \therefore \quad y = 40$$

Hence, the speeds of two cars are 60 km/h and 40 km/h respectively.

Q17. The sum of a two digit number and the number formed by interchanging its digits is 110. It 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

Sol. Let the digits at unit and tens places be x and y respectively.

Then, number = 10y + x ----(i)

Number formed by the interchanging the digits = 10x + y

According to the given condition, we have

$$(10y + x) + (10x + y) = 110 \quad \Rightarrow \quad 11x + 11y = 110$$

$$\Rightarrow \quad x + y - 10 = 0$$

Again, according to question, we have

$$(10y + x) - 10 = 5(x + y) + 4 \quad \Rightarrow \quad 10y + x - 10 = 5x + 5y + 4$$

$$\Rightarrow \quad 10y + x - 5x - 5y = 4 + 10$$

$$5y - 4x = 14 \text{ or } 4x - 5y + 14 = 0$$

By using cross-multiplication, we have

$$\frac{x}{1 \times 14 - (-5) \times (-10)} = \frac{-y}{1 \times 14 - 4 \times (-10)} = \frac{1}{1 \times (-5) - 1 \times 4}$$

$$\Rightarrow \quad \frac{x}{14 - 50} = \frac{-y}{14 + 40} = \frac{1}{-5 - 4} \quad \Rightarrow \quad \frac{x}{-36} = \frac{-y}{-54} = \frac{1}{-9}$$

$\Rightarrow \quad x = \frac{-36}{-9}$ and $y = \frac{-54}{-9} = \frac{1}{-9} \quad \Rightarrow \quad x = 4$ and $y = 6$

Putting the value of x and y in equation (i), we get

Number 10 x 6 + 4 = 64

Q18. Students of a class are made to stand in rows. If one student is extra in each row, there would be 2 rows less. If one student is less in each row, there would be 3 rows more. Find the number of students in the class.

Sol. Let total number of rows be y

and total number of students in each row be x

$\therefore$ Total number of students = xy

Case I : If one student is extra in each row, there would be two rows less.

Now, number of rows = (y – 2)

Number of students in each row = (x + 1)

Total number of students = Number of rows x Number of students in each row

$xy = (y - 2)(x + 1) \quad \Rightarrow \quad xy = xy + y - 2x - 2$

$\Rightarrow \quad xy - xy - y + 2x = -2 \quad \Rightarrow \quad 2x - y = -2$ ---(i)

Case II : If one student is less in each row, there would be 3 rows more.

Now, number of rows = (y + 3)

and number of students in each row = (x – 1)

Total number of students = Number of rows x Number of students in each row

$\therefore \quad xy = (y + 3)(x - 1) \quad \Rightarrow \quad xy = xy - y + 3x - 3$

$xy - xy + y - 3x = -3 \quad \Rightarrow \quad -3x + y = -3$ ---(ii)

On adding equations (i) and (ii), we have

$$\begin{array}{r} 2x - y = -2 \\ -3x + y = -3 \\ \hline -x = -5 \end{array}$$

or $x = 5$

Putting the value of x in equation (i), we get

$2(5) - y = -2 \quad \Rightarrow \quad 10 - y = -2$

$-y = -2 - 10 \quad \Rightarrow \quad -y = -12$

or $\quad y = 12$

$\therefore$ Total number of students in the class = 5 x 12 = 60

Q19. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol. Let the ages of Ani and Biju be x and y years respectively. Then $x - y = \pm 3$

Age of Dharam = 2x years

Age of Cathy = $\frac{y}{2}$ years

Clearly, Dharam is older than Cathy.

$\therefore \quad 2x - \frac{y}{2} = 30 \quad \Rightarrow \quad \frac{4x - y}{2} = 30 \quad \Rightarrow 4x - y = 30$

Thus, we have following two systems of linear equations

$x - y = 3$ ---(i)

$4x - 4 = 60$ ---(ii)

and $x - y = -3$ ---(iii)

$4x - y = 60$ ---(iv)

Subtracting equation (i) from (ii), we get

$$\begin{array}{r} 4x - y = 60 \\ x - y = 3 \\ - \quad + \quad - \\ \hline 3x = 57 \end{array} \quad \Rightarrow \quad x = 19$$

Putting x = 19 in equation (iii) from (iv)

$$\begin{array}{r} 4x - y = 60 \\ x - y = 3 \\ - \quad + \quad + \\ \hline 3x = 63 \end{array} \quad \Rightarrow \quad x = 21$$

Putting x = 21 in equation (iii), we get

$21 - y = -3 \quad \Rightarrow \quad y = 24$

Hence, age of Ani = 19 years and age of Biju = 16 years

or age of Ani = 21 years and age of Biju = 24 years

CHAPTER - 4 : QUADRATIC EQUATIONS

Q20. If ad = bc, then prove that the equation

$(a^2 + b^2) x^2 + 2(ac + bd) x = (c^2 + d^2) = 0$ has no real roots

Sol. The given quadratic equation is $(a^2 + b^2) x^2 + 2(ac + bd) x = (c^2 + d^2) = 0$

$D = b^2 - 4ac$

$= 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= -4(a^2d^2 + b^2c^2 - 2abcd) = -4(ad - bc)^2$

Since ad $\neq$ bc

There D < 0

Hence, the equation has no real roots.

Q21. Solve for $x : \sqrt{3}x^2 - 2x - 8\sqrt{3} = 0$

Sol. $x : \sqrt{3}x^2 - 2x - 8\sqrt{3} = 0$

By mid term splitting

$\Rightarrow \quad \sqrt{3}x^2 - 6x + 4x - 8\sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}x(x - 2\sqrt{3}) + 4(x - 2\sqrt{3}) = 0 \qquad \Rightarrow \quad (x - 2\sqrt{3})(\sqrt{3}x + 4) = 0$

$\Rightarrow$ Either $(x - 2\sqrt{3}) = 0$ or $(\sqrt{3}x + 4) = 0$

$\Rightarrow \quad x = \frac{-4}{\sqrt{3}},\ 2\sqrt{3}$

Q22. Find the roots of the following equation :

$$\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}; x \neq -3, 6$$

Sol. Given $\frac{1}{x+3} - \frac{1}{x-6} = \frac{9}{20}; x \neq -3, 6$

$\Rightarrow \quad \frac{(x-6)-(x+3)}{(x+3)(x-6)} = \frac{-}{20} \qquad \Rightarrow \quad \frac{-9}{(x+3)(x-6)} = \frac{9}{20}$

$\Rightarrow \quad (x + 3)(x - 6) = -20 \qquad$ or $\qquad x^2 - 3x + 2 = 0$

$\Rightarrow \quad x^2 - 2x - x + 2 = 0 \qquad \Rightarrow \quad x(x - 2) - 1(x - 2) = 0$

$\Rightarrow \quad (x - 1)(x - 2) = 0 \qquad \Rightarrow \quad x = 1$ or $x = 2$

Both x = 1 and x = 2 are satisfying the given equation. Hence, x = 1, 1, 2 are the solutions of the equation.

Q23. Solve for $x : \frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x=3)} = 0, x \neq 3, -\frac{3}{2}$

Sol. $x : \frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x=3)} = 0, x \neq 3, -\frac{3}{2}$

$\Rightarrow$ $2x(2x + 3) + (x – 3) + (3x + 9) = 0$

$\Rightarrow$ $4x^2 + 10x + 6 = 0$ $\Rightarrow$ $2x^2 + 5x + 3 = 0$

$\Rightarrow$ $(x + 1) (2x + 3) = 0$ $\Rightarrow$ $x = –1, x = -\frac{3}{2}$

But $x = -\frac{3}{2}$ $\therefore$ $x = –1$

Q24. Solve for $x : \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$

Sol. $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$ $\Rightarrow$ $\frac{(x-3) = (x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$

$\Rightarrow$ $3(x – 3 + x – 1) = 2(x – 1)(x – 3)$ $\Rightarrow$ $3(2x – 4) = 2(x – 1)(x –2)(x – 3)$

$\Rightarrow$ $3 \text{ x } 2(x – 2) = 2(x – 1)(x – 3)$ $\Rightarrow$ $3 = (x – 1)(x – 3)$ i.e., $x^2 – 4x = 0$

$\Rightarrow$ $x(x – 4) = 0$ $\therefore$ $x = 0, x = 4$

Q25. Find the value fo p for which the quadratic equation
$(2p + 1)x^2 –(7p + 2)x +(7p – 3) = 0$ has equal roots. Also find these roots.

Sol. Since the quadratic equation has equal roots, D = 0

i.e., $b^2 – 4ac = 0$

In $(2p + 1)x^2 – (7p + 2)x + (7p – 3) = 0$

Here, $a = (2p + 1), b = –(7p + 2), c = (7p –3)$

$\therefore$ $(7p + 2)^2 – 4(2p + 1), c = (7p – 3)$

$\Rightarrow$ $49p^2 + 4 – 28p – (8p + 4)(7p – 3) = 0$

$\Rightarrow$ $49p^2 + 4 + 28p – 56p^2 + 28p – 28 p + 12 = 0$

$\Rightarrow$ $–7p^2 + 24p + 16 = 0$ $\Rightarrow$ $7p^2 – 24p – 16 = 0$

$\Rightarrow$ $7p^2 – 28p + 4p – 16 = 0$ $\Rightarrow$ $7p (p – 4) + 4 (p – 4) = 0$

$\Rightarrow$ $(7p + 4)(p – 4) = 0$ $\Rightarrow$ $p = -\frac{4}{7}$ or $p = 4$

Q26. A train travels 360 km at a uniform speed. If the speed has been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. Let the uniform speed of the train be x km/h.

Then, time taken to cover 360 km $=\frac{360}{x}$h

Now, new increased speed = (x = 5) km/h

So, time taken to cover 360 km $=\frac{360}{x+5}$h

According to question, $\frac{360}{x}=\frac{360}{x+5}=1$

$\Rightarrow \quad 360\left(\frac{1}{x}-\frac{1}{x+5}\right)=1 \qquad \Rightarrow \quad \frac{360(x+5-x)}{x(x+5)}=1$

$\Rightarrow \quad \frac{360+5}{x(x+5)}=1 \qquad \Rightarrow \quad 1800 = x^2 + 5x$

$\therefore \quad x^2 + 5x - 1800 = 0 \qquad \Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0 \qquad \Rightarrow \quad (x + 45)\ (x - 40) = 0$

Either x + 45 = 0 or x = 40 = 0

$\therefore \quad x = -45$ or $x = 40$

But x cannot be negative, so $x \neq -45$

Therefore, x = 40

Hence, the uniform speed of train is 40 km/h.

Q27. The sum of the areas of two squares is 468m²**. If the difference of their perimeters is 24m, find the sides of the two squares.**

Sol. Let x be the length of the side of first square and y be the length of side of the second square.

Then, $x^2 + y^2 = 468$ ---(i)

Let x be the length of the side of the bigger square.

$4x - 4y = 24$

$\Rightarrow \quad x - 4 = 6$ or $x = y + 6$ ---(ii)

Putting the value of x in terms of y from equation (ii), in equation (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow \quad y^2 + 12y + 36 + y^2 = 468$ or $2y^2 + 12y - 432 = 0$

$\Rightarrow \quad y^2 + 6y - 216 = 0 \qquad \Rightarrow \quad y^2 + 18y - 12y - 216 = 0$

$\Rightarrow$ y(y + 18) –12(y + 18) = 0 $\Rightarrow$ (y + 18) (y – 12) = 0

Either y + 18 = 0 or y – 12 = 0

$\Rightarrow$ y = –18 or y = 12

But, sides cannot be negative, so y = 12

Therefore, x = 12 + 6 = 18

Hence, sides of two squares are 18m and 12m.

Q28. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

Sol. Let the digit at tens place be x.

Then, digit at unit place $=\frac{18}{x}$

$\therefore$ Number $=10x+\frac{18}{x}$

and number obtained by interchaning the digits $=10\ x\frac{18}{x}+x$

According to the question,

$$\left(10x+\frac{18}{x}\right)-63=10\ x\frac{18}{x}+x \qquad \Rightarrow \qquad =\left(10x+\frac{18}{x}\right)-\left(10\ x\frac{18}{x}+x\right)=63$$

$$\Rightarrow \quad 10x+\frac{18}{x}-\frac{180}{x}-x=63 \qquad \Rightarrow \quad 9x-\frac{162}{x}-63=0$$

$\Rightarrow$ $9x^2 - 63x - 162 = 0$ $\Rightarrow$ $x^2 - 7x - 18 = 0$

$\Rightarrow$ $x^2 - 9x + 2x - 18 = 0$ $\Rightarrow$ x(x – 9) + 2(x – 9) = 0

$\Rightarrow$ (x – 9) (x + 2) = 0 $\Rightarrow$ x = 9 or x = –2

$\Rightarrow$ x = 9 [$\therefore$ a digit can never be negative]

Hence, the required number $=10\ x\ 9+\frac{18}{9}=92$

Q29. If twice the area of a smaller square is subtracted from the area of a larger square; the result is 14cm^2. However, if twice the area of the larger square is added to three times the area of the smaller square, the result is 203cm^2. Determine the sides of the two squares.

Sol. Let the sides of the larger and smaller squares be x and y respectively. Then

$x^2 - 2y^2 = 14$ ---(i)

and $2x^2 + 3y^2 = 203$ ---(ii)

Operating (ii) –2 x (i), we get

$$2x^2 + 3y^2 - (2x^2 - 4y^2) = 203 - 2 \times 14$$

$\Rightarrow$ $2x^2 + 3y^2 - 2x^2 - 4y^2 = 203 - 28$

$\Rightarrow$ $7y^2 = 172$ $\Rightarrow$ $y^2 = 25$ $\Rightarrow$ $y = \pm 5$

$\Rightarrow$ $y = 5$ [$\because$ Side cannot be negative]

By putting the value of y in equation (i), we get

$x^2 - 2 \times 5 = 14$ $\Rightarrow$ $x^2 - 50 = 14$ or $x^2 = 64$

$\therefore$ $x = \pm 8$ or $x = 8$

$\therefore$ Sides of the two squares are 8cm and 5cm.

Q30. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Sol. Let x be the total number of camels.

Then, number of camels in the forest $= \frac{x}{4}$

Number of camels on mountains $= 2\sqrt{x}$

and number of camels on the bank of river = 15

Now, by hypothesis, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x \qquad \Rightarrow 3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x}$ = y, then $x = y^2$

$\Rightarrow$ $3y^2 - 8y - 60 = 0$ $\Rightarrow$ $3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow$ $3y(y - 6) + 10(y - 6) = 0$ $\Rightarrow$ $(3y + 10)(y - 6) = 0$

$\Rightarrow$ $y = 6$ or $y = -\frac{10}{3}$

Now, $y = -\frac{10}{3}$ $\Rightarrow$ $x = \left(-\frac{10}{3}\right)^2 = \frac{100}{9}$ ($\because$ $x = y^2$)

But the number of camels cannot be a fraction.

$\therefore$ y = 6 $\Rightarrow$ $x = 6^2 = 36$

Hence, the number of camels = 36

Q31. Solve the following equation :

$9x^2 - 9(a + b)x + [2a^2 + 5ab + 2b^2] = 0$

Sol. Consider the equation $9x^2 - 9(a + b)x + [2a^2 + 5ab + 2b^2] = 0$

Now comparing with $Ax^2 + Bx + C = 0$, we get

$A = 9$, $B = -9(a + b)$ and $C = [2a^2 + 5ab + 2b^2]$

Now discriminant,

$D = B^2 = 4AC$

$= \{-9(a + b)\}^2 - 4 \text{ x } 9\{2a^2 + 5ab + 2b^2) = 9^2 (a + b)^2 - 4 \text{ x } 9 (2a^2 + 5ab + 2b^2)$

$= 9 \{9 (a + b)^2 - 4 (2a^2 + 5ab + 2b^2)\} = 9 \{9a^2 + 9b^2 + 18ab - 8a^2 - 20ab - 8b^2$

$= 9 \{a^2 + b^2 - 2ab\} = 9 (a - b)^2$

Now using the quadratic formula.

$$x = \frac{-B \pm \sqrt{D}}{2A}, \text{we get } x = \frac{9(a+b) \pm \sqrt{9(a-b)^2}}{2x9}$$

$$\Rightarrow \quad x = \frac{9(a+b) \pm 3(a-b)}{2 \text{ x } 9} \quad \Rightarrow \quad x = \frac{3(a+b) \pm 3(a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(3a+3b)+(a-b)}{6} \quad \text{and} \quad x = \frac{(3a+3b)-(a-b)}{6}$$

$$\Rightarrow \quad x = \frac{(4a+2b)}{6} \quad \text{and} \quad x = \frac{(2a+4b)}{6}$$

$$\Rightarrow \quad x = \frac{2a+b}{3} \quad \text{and} \quad x = \frac{a+2b}{3}$$ are required solutions.

Q32. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?

Sol. Let, time taken by faster tap to fill the tank be x hours

Therefore, time taken by slower tap to fill the tank = (x + 3) hours

Since the faster tap takes x hours to fill the tank.

$\therefore$ Portion of the tank filled by the faster tap in one hour $= \frac{1}{x}$

Portion of the tank filled by the slower tap in one hour $= \frac{1}{x+3}$

Portion of the tank filled by the two tap together in one hour $= \frac{1}{\frac{40}{13}} = \frac{13}{40}$

According to question

$\Rightarrow \quad \frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$ $\qquad \Rightarrow \quad \frac{x+3+x}{x(x+3)} = \frac{13}{40}$

$\Rightarrow \quad 40(2x + 3) = 13x\ (x + 3)$ $\qquad \Rightarrow \quad 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$ $\qquad \Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad 13x\ (x - 5) + 24\ (x - 5) = 0$ $\qquad \Rightarrow \quad (x - 5)\ (13x + 24) = 0$

Either $x - 5 = 0$ or $13x + 24 = 0$

$\Rightarrow \quad x - 5 = 0 \quad$ or $\quad x = \frac{-24}{13}$

$\Rightarrow \quad x = 5 \qquad [\because$ x cannot be negative]

Hence, time taken by faster tap to fill the tank = x = 5 hours.

and time taken by slower up tap = x + 3 = 5 + 3 = 8 hours

CHAPTER - 5 : COORDINATE GEOMETRY

Q33. Determine, if the points (1, 5), (2, 3) and (–2, –11) and collinear.

Sol. Let A (1, 5), B (2, 3) and C (–2, –11) be the given points. Then we have

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5}$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{16+196} = \sqrt{4 \text{ x } 53} = 2\sqrt{53}$$

$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{9+256} = \sqrt{265}$$

Clearly, $\quad \text{Ab} + \text{BC} \neq \text{AC}$

$\therefore$ A, B, C are not collinear

Q34. Find the value of y for which the distance between the points P(2, –3) and Q(10, y) is 10 units.

OR

A line segment is of length 10 units. If the coordinates of its one end are (2, –3) and the abscissa of the other end is 10, find its ordinate.

Sol. We have, PQ = 10

$\Rightarrow \quad \sqrt{(10-2)^2+(y+3)^2}=10$

Squaring both sides, we have

$\Rightarrow$ $(8)^2 + (y + 3)^2 = 100$ $\Rightarrow$ $(y + 3)^2 = 100 – 64$

$\Rightarrow$ $(y + 3)^2 = 36$ or $y + 3 = \pm 6$

$\Rightarrow$ $y + 3 = 6, y + 3 = –6$ or $y = 3, y = –9$

Hence, value of y are –9 and 3.

Q35. Find the area of a rhombus if its vertices (3, 0), (4, 5), (–1, 4) and (–2, –1) are taken in order.

Sol. Let A(3, 0), B (4, 5), C (–1, 4) and D(–2, –1) be the vertices of a rhombus.

Therefore, its diagonals

$$AC=\sqrt{(-1-3)^2+(4-0)^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

and $$BD=\sqrt{(-2-4)^2+(-1-5)^2}=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$$

$\therefore$ Area of rhombus ABCD $=\frac{1}{2}$ x (Product of length of diagonals)

$$=\frac{1}{2}\text{ x AC x BD}=\frac{1}{2}\text{ x }4\sqrt{2}\text{ x }6\sqrt{2}=24\text{ sq units}$$

Q36. Find the area of the triangle whose vertices are : (–5, –1), (3, –5), (5, 2)

Sol. Let $A(x_1, y_1) = (–5, –1)$, $B(x_2, y_2) = (3, –5)$, $C (x_3, y_3) = (5, 2)$

$\therefore$ area of $\Delta ABC=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$=\frac{1}{2}[-5(-5-2)+3(2+1)+5(-1+5)]$$

$$=\frac{1}{2}[35+9+20)=\frac{1}{2}\text{ x }64=32\text{ sq units}$$

Q37. If the point P (k –1, 2) is equidistant from the points A (3, k) and B (k, 5), find the values of k.

Sol. Let the given line segment be divided by point Q. Since P is equidistant from A and B,

AP = BP or AP2 = BP2

$[3 –(k–1)]^2 + (k – 2)^2 = [k – (k – 1)]^2 + (5 – 2)^2$

$(3 – k + 1)^2 + (k –2)^2 = (k – k +1)^2 + (3)^2$

$(4 – k)^2 + (k – 2)^2 = (1)^2 + (3)2$ $\Rightarrow$ $16 + k^2 – 8k + k^2 + 4 – 4k = 1 + 9$

$2k^2 – 12k + 20 = 10$ $\Rightarrow$ $k^2 – 6k + 10 = 5$

$k^2 - 6k + 5 = 0 \qquad \Rightarrow k^2 - 5k - k + 5 = 0$

$k(k - 5) - 1(k - 5) = 0 \qquad \Rightarrow k = 1 \text{ or } k = 5$

Q38. Find the value of k if the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear.

Sol. Points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear

$\therefore$ Area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}[k+1)(2k = 3 - 5k) + 3k(5k - 2k) + (5k-1)\{2k - (2k+3)\}] = 0$$

$$\Rightarrow \frac{1}{2}[k+1)(-3k+3) + 3k(3k) + (5k-1)(2k - 2k - 3)] = 0$$

$$\Rightarrow \frac{1}{2}[-3k^2 + 3k - 3 + 9k^2 - 15k + 3] = 0$$

$$\Rightarrow \frac{1}{2}[6k^2 - 15k + 6] = 0 \qquad \Rightarrow 6k^2 - 15k + 6 = 0$$

$$\Rightarrow 2k^2 - 5k + 2 = 0 \qquad \Rightarrow 2k^2 - 4k - k + 2 = 0$$

$\Rightarrow (k - 2)(2k - 1) = 0$

If k – 2 = 0, then k = 2

If 2k – 1 = 0, then $k = \frac{1}{2}$

$\therefore \quad k = 2, \frac{1}{2}$

Q39. Determine the ratio in which the lines 2x + y – 4 = 0 divides the line segment joining the points A(2, –2) and B(3, 7).

Sol. Let $P(x_1, y_1)$ be common points of both lines and divide the line segment joining A(2, –2) and B(3, 7) in ration k:1.

$$\therefore \quad x_1 = \frac{3k+2}{k+1} \text{ and } y_1 = \frac{7k + 1(-2)}{k+1} = \frac{7k-2}{k+1}$$

Since, point (x_1, y_1) lies on the line 2x + y = 4

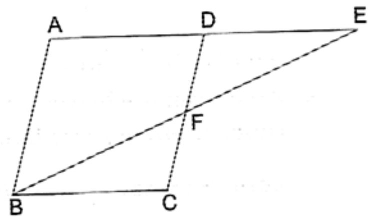
$$\therefore \qquad 2\left(\frac{3k+2}{k+1}\right)\left(\frac{7k-2}{k+1}\right)=4 \qquad \Rightarrow \qquad \frac{6k+4+7k-2}{k+1}=4$$

or 13k + 2 = 4k + 4 or 9k = 2 or $k=\frac{2}{9}$

So, required ratio is $\frac{2}{9}$: 1 or 2 : 9

CHAPTER - 6 : TRIANGLES

Q40. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$



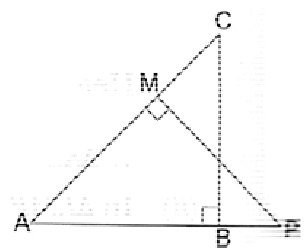
Sol. In ΔABE and ΔCFB, we have

$\angle AEB = \angle CBF$ (Alternate angles)

$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\therefore \quad \Delta ABE \sim \Delta CFB$ (By AA criterion of similarity)

Q41. In ABC and AMP are two right triangles right-angled at B and M respectively. Prove that :



(i) $\Delta ABC \sim \Delta AMP$ **(ii)** $\frac{CA}{PA}=\frac{BC}{MP}$

Sol. (i) In ΔABC and ΔAMP, we have

$\angle ABC = \angle AMP = 90°$ (Given)

And, $\angle BAC = \angle MAP$ (Common angle)

$\therefore \quad \Delta ABC \sim \Delta AMP$ (By AA criterion of similarity)

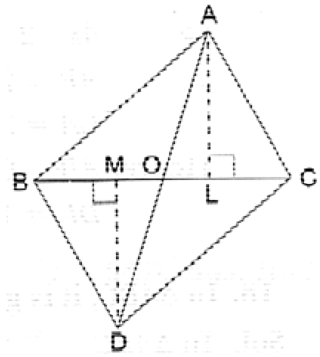
(ii) As $\Delta ABC \sim \Delta AMP$ (Proved above)

$\therefore \quad \frac{CA}{PA}=\frac{BC}{MP}$ (Sides of similar triangles are proportional)

Q42. In the given ΔABC and ΔDBC **are on the same base BC. If AD intersects BC at O.**

Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$

Sol. Given: ΔABC and ΔDBC are on the same base BC and AD intersects BC at O.



To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction : Draw AL $\perp$ BC and DM $\perp$ BC

Proof : In ΔALO and ΔDMO, we have

$\angle$ ALO = DMO = 90° and

$\angle$ AOL = $\angle$ DOM (Vertically opposite angles)

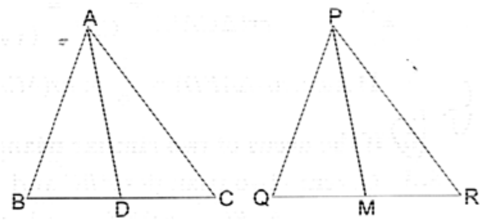
$\therefore$ $\Delta ALO \sim \Delta DMO$ (By AA - Similarity)

$\Rightarrow$ $\frac{AL}{DM} = \frac{AO}{DO}$ ---(i)

$\therefore$ $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AL}{\frac{1}{2} BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ (Using (i))

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Q43. If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.



Sol. In ΔABC and ΔPQM we have

$\angle$ B = $\angle$ Q ($\because \Delta ABC \sim \Delta PQR$) ---(i)

$\frac{AB}{PQ} = \frac{BC}{QR}$ ($\because \Delta ABC \sim \Delta PQR$)

$\Rightarrow$ $\frac{AB}{PQ} = \frac{\frac{1}{2} BC}{\frac{1}{2} QR}$ $\Rightarrow$ $\frac{AB}{PQ} = \frac{BD}{QM}$ ---(ii)

[Since Ad and PM are the medians of ΔABC and ΔPQR respectively]

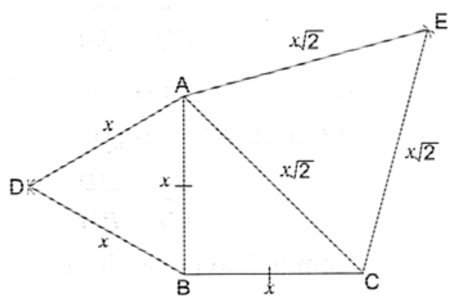
From (i) and (ii), it is proved that

$\Delta ABD \sim \Delta PQM$ (By SAS criterion of similarity)

$\Rightarrow \quad = \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \qquad \Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$

Q44. Prove that the area of an equilaterial triangle described on a side of a right-angled isosceles triangle is half the area of the equilateral triangle described on its hypotenuse.

Sol. Given: ΔABC in which $\angle ABC = 90°$ and AB = BC. ΔABD and ΔCAE are equilaterial triangles.



To Prove : ar(Δ ABD) = $\frac{1}{2}$ x ar(Δ CAE)

Proof : Let AB = BC = x units

$\therefore$ hyp. $CA = \sqrt{x^2 + x^2} = x\sqrt{2}$ units

Each of the Δ ABD and Δ CAE being equilaterial, has each angle equal to 60°

$\therefore \quad \Delta ABD \sim \Delta CAE$

But, the ratio of the area of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABD)}{ar(\Delta CAE)} = \frac{AB^2}{CA^2} = \frac{x^2}{(x\sqrt{2})^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

Hence, $ar(\Delta ABD) = \frac{1}{2}$ x $ar(\Delta CAE)$

Q45. If the area of two similar triangles are equal, prove that they are congruent.

Sol. Given : Two triangles ABC and DEF, such that

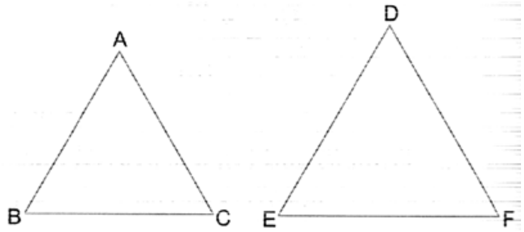
Δ ABC ~ Δ DEF and area (Δ ABC) = area (Δ DEF)

To prove: $\Delta ABC \cong \Delta DEF$

Proof : Δ ABC ~ Δ DEF

$\Rightarrow \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



Now, ar(Δ ABC) = ar(Δ DEF) (Given)

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = 1$ ---(i)

and $\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)}$ ($\because \Delta$ ABC ~ Δ DEF) ---(ii)

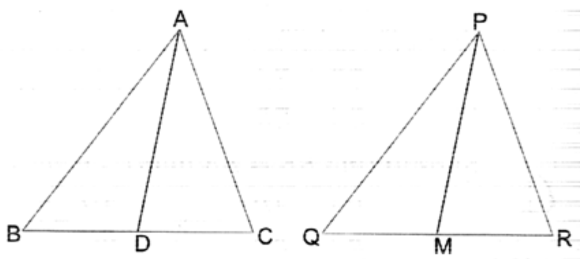
From (i) and (ii), we have

$$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} = 1 \quad \Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$$

Hence, $\Delta ABC \cong \Delta DEF$ (By SSS criterion of congruency)

Q46. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Sol. Let Δ ABC and Δ PQR be two similar triangles. AD and PM are the medians of Δ ABC and Δ PQR respectively.



To prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof : Since Δ ABC ~ Δ PQR

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \text{---(i)}$$

In Δ ABD and Δ PQM

$$\frac{AB}{PQ} = \frac{BD}{QM} \qquad \left(\because \frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \right)$$

and $\angle$B = $\angle$Q ($\because$ Δ ABC ~ Δ PQR)

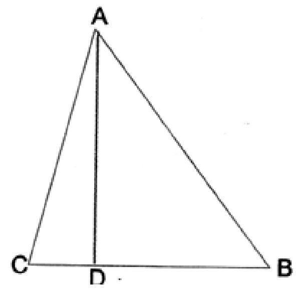
Hence, Δ ABD ~ Δ PQM (By SAS similarity criterion)

$$\frac{AB}{PQ} = \frac{AD}{PM} \quad \text{---(ii)}$$

From (i) and (ii), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

Q47. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.



Sol. We have, DB = 3CD

$\Rightarrow$ BC = BD + CD

$\therefore$ $CD = \frac{1}{4}BC$

and $DB = 3CD = \frac{3}{4}BC$

Now, in right-angled triangle ABD using Pythagoras Theorem we have

$AB^2 = AD^2 + DB^2$ ---(i)

Again, in right-angled triangle ΔADC, we have

$AC^2 = AD^2 + CD^2$ ---(ii)

Subtracting (ii) from (i), we have

$AB^2 – AC^2 = DB^2 - CD^2$

$$\Rightarrow \quad AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2 = \frac{8}{16}BC^2$$

$$\Rightarrow \quad AB^2 - AC^2 = \frac{1}{2}BC^2$$

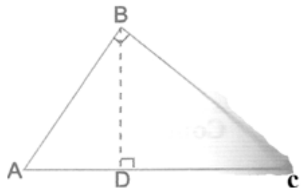
$\therefore$ $2AB^2 – 2AC^2 = BC^2$ $\Rightarrow$ $2AB^2 = 2AC^2 + BC^2$

Q49. Provide that, in a right triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Using the above, do the following :

Prove that, in a ΔABC if AD is perpendicular to BC, then $AB^2 + CD^2 = AC^2 + BD^2$

Sol. Given: A right triangle ABC right-angled at B.



To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In Δ ADB and Δ ABC

$\angle A = \angle A$ (Common)

$\angle ADB = \angle ABC$ (Both 90°)

$\therefore$ $\Delta ADB \sim \Delta ABC$ (AA similarity criterion)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

or $AD\ AC = AB^2$ ---(i)

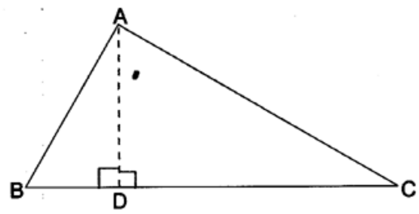
In Δ BDG and Δ ABC

$\angle C = \angle C$ (Common)

$\angle BDC = \angle ABC$ (Each 90°)

$\Delta BDC \sim \Delta ABC$ (AA similarity)

So, $\frac{CD}{BC} = \frac{BC}{AC}$ or; $CD.\ AC = BC^2$ ---(ii)



Adding (i) and (ii), we get

$AD\ .\ AC + CD\ .\ AC = AB^2 + BC^2$

or, $AC\ (AD + CD) = AB^2 + BC^2$

or, $AC\ .\ AC = AB^2 + BC^2$

or, $AC^2 = AB^2 + BC^2$ (From figure)

Second Part :

In Fig, As $AD \perp BC$

Therefore, $\angle ADB = \angle ADC = 90°$

By Pythagoras Theorem, we have

$AB^2 = AD^2 + BD^2$ ---(i)

$AC^2 = AD^2 + DC^2$ ---(ii)

Subtracting (ii) from (i)

$AB^2 - AC^2 = AD^2 + BD^2 - (AD^2 + DC^2)$

$AB^2 - AC^2 = BD^2 - DC^2$

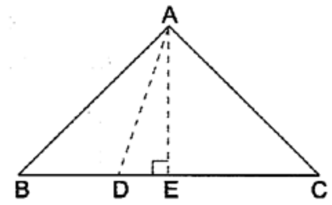
$\Rightarrow$ $AB^2 + DC^2 = BD^2 + AC^2$

Q50. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Sol. Given : An equilateral triangle ABC and D be a point on BC such that $BD = \frac{1}{3}BC$.

To prove: $9AD^2 = 7AB^2$

Construction : Draw $AE \perp BC$. Join AD.



Proof : Δ ABC is an equilateral triangle and $AE \perp BC$

BE = EC

Thus, we have

$$BD = \frac{1}{3}BC \text{ and } DC = \frac{2}{3}BC \text{ and } BE = EC = \frac{1}{2}BC$$

In Δ AEB

$AE^2 + BE^2 - AB^2$ [Using Pythagoras Theorem)

$AE^2 = AB^2 - BE^2$

$AD^2 - DE^2 = AB^2 - BE^2$ [$\because$ In Δ AED, $AD^2 = AE^2 + DE^2$]

$AD^2 = AB^2 - BE^2 + DE^2$

$$AD^2 = AB^2 - \left(\frac{1}{2}BC\right)^2 + (BE^2 - BD)^2$$

$$AD^2 = AB^2 - \left(\frac{1}{2}BC\right)^2 + \left(\frac{1}{2}BC - \frac{1}{3}BC\right)^2$$

$$AD^2 = AB^2 - \frac{1}{2}BC^2 + \left(\frac{BC}{6}\right)^2 \quad \Rightarrow \quad AD^2 = AB^2 - BC^2\left(\frac{1}{4} - \frac{1}{36}\right)$$

$$AD^2 = AB^2 - BC^2\left(\frac{8}{36}\right) \quad \Rightarrow \quad 9AD^2 = 9AB^2 - 2BC^2$$

$9AD^2 = 9AB^2 - 2AB^2$ [$\because$ AB = BC]

$9AD^2 = 7AB^2$

Q51. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC at L and AD produced to E. Prove that EL = 2BL

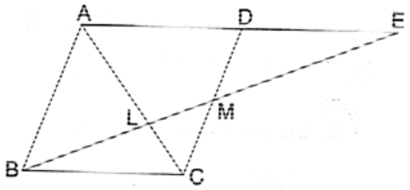
Sol. In ΔBMC and ΔEMD, we have

MC = MD [$\because$ M is the mid-point of CD]

$\angle$CMB = $\angle$DME [Vertically opposite angles]

and $\angle$MBC = $\angle$MED [Alternate angles]

So, by AAS criterion of congruence, we have



$$\Delta BMC \cong \Delta EMD$$

$\Rightarrow$ BC = DE [CPCT]

Also, BC = AD [$\because$ ABCD is a parallelogram]

Now, in ΔAEL and ΔCBL, we have

$\angle$ALE = $\angle$CLB [Vertically opposite angles]

$\angle$EAL = $\angle$BCL [Alternate angles]

So, by AA criterion of similarity of triangles, we have

ΔAEL ~ ΔCBL

$$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{CB}$$

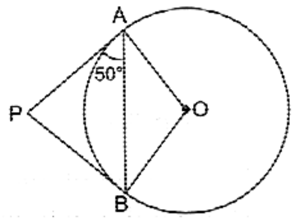
$$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC}$$ [$\because$ Ae = AD + DE = BC + BC = 2BC]

$$\Rightarrow \quad \frac{EL}{BL} = 2$$

$\Rightarrow$ EL = 2BL

CHAPTER - 7 : CIRCLES

Q52. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle$PAB = 50°, then find $\angle$AOB.

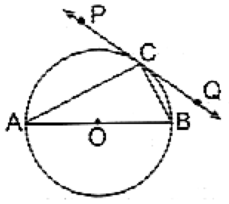


Sol. $\because$ PA = PB $\Rightarrow$ $\angle$BAP = $\angle$ABP = 50°

$\because$ $\angle$ABP = 180° – 50° – 50° = 80°

$\because$ $\angle$AQB = 180° – 80° = 100°

Q53. In fig. PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle$CAB = 30°, find $\angle$PCA.



Sol. $\angle ACB = 90°$ (Angle in the semicircle)

$\angle CAB = 30°$ (given)

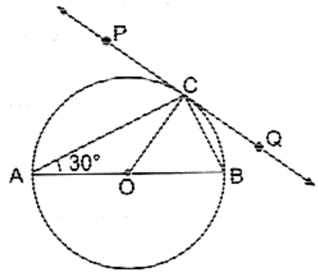
In ΔABC,

$90° + 30°$

$\Rightarrow$ $\angle ABC = 60°$

Now, $\angle PCA = \angle ABC$ (Angles in the alternate segment)

$\therefore$ $\angle PCA = 60°$



OR

Construction : Join O to C.

$\angle PCO = 90°$ [$\because$ Line joining centre to point of contact is perpendicular to PQ]

In ΔAOC, OA = OG [Radii of circle]

$\therefore$ $\angle OAC = \angle OCA = 30°$ [Equal sides have equal opp. angles]

Now, $\angle PCA = \angle PCO - \angle ACO$

$= 90° - 30° = 60°$

Q54. A quadrilaterial ABCD is drawn to circumscribe a circle (Fig.) Prove that AB + CD = AD + BC.

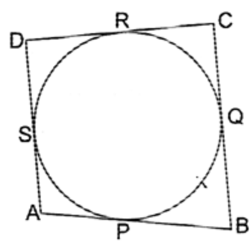
OR

A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA

Sol. Since lenghts of two tangents drawn from an external point of circle are equal.

Therefore, AP = AS, BP = BQ and DR = DS

CR = CQ [Where P, Q, R and S are the points of contact]



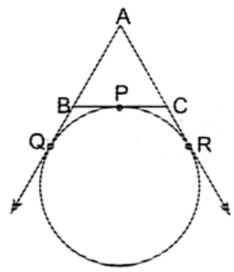
Adding all these, we have

(AP + BP) + (CR + RD) = (BQ + CQ) + (DS + AS)

$\Rightarrow$ AB + CD = BC + DA

Q55. A circle is touching the side BC of ΔABC at P and touching AB and AC prodced at Q and R respectively. Prove that AQ = $\frac{1}{2}$ (Perimeter of ΔABC).

Sol. Since tangents from an exterior point to a circle are equal in length.



$\therefore$ BP = BQ [Tangents from B] ---(i)

CP = CR [Tangents from C] ---(ii)

and AQ = AR [Tangents from A] ---(iii)

From (iii), we have

AQ = AR $\Rightarrow$ AB + BQ = AC + CR

$\Rightarrow$ AB + BP = AC + CP [Using (i) and (ii)] ----(iv)

Now, perimeter of ΔABC = AB + BC + AC

= AB + (BP + PC) + AC

= (AB + BP) + (AC + PC)

= 2(AB + BP) [Using (iv)]

= 2(AB + BQ) = 2AQ [Using (i)]

$\therefore$ AQ = $\frac{1}{2}$(Perimeter of ΔABC)

Q56. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

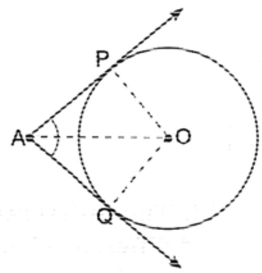
Sol. Given : AP and AQ are two tangents from a point A to a circle C (O, r)

To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : In order to prove that AP = AQ, we shall first prove that ΔOPA $\cong$ ΔOQA.

Through the point of contact.



$\therefore$ OP $\perp$ AP and OQ $\perp$ AQ

$\Rightarrow$ $\angle$OPA = $\angle$OQA = 90°

Now, in right triangles OPA and OQA, we have

OP = OQ [Radii of a circle]

$\angle$OPA = $\angle$OQA [Each 90°]

and OA = OA [Common]

So, by RHS-criterion of congruence, we get

ΔOPA $\cong$ OQA $\Rightarrow$ AP = AQ [CPCT]

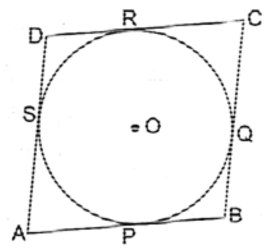
Hence, lengths of two tangents from an external point are equal.

Q57. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Let ABCD be a parallelogram such that its sides touch a circle with centre O.

We know that the tangents to a circle from an exterior point are equal in length.

Therefore, we have



AP = AS [Tangents from A] ---(i)

BP = BQ [Tangents from B] ---(ii)

CR = CQ [Tangents from C] ---(iii)

And DR = DS [Tangents from D] ---(iv)

Adding (i), (ii), (iii) and (iv), we have

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

AB + AB = BC + BC [$\because$ ABCD is a parallelogram $\therefore$ AB =CD, BC=DA]

2AB = 2BC $\Rightarrow$ AB = BC

Thus, AB = BC = CD = AD

Hence, ABCD is a rhombus.

Q58. In Fig, XY and X'Y' are two parallel tangents to a circle with centre O and another tangents AB with point of contact C intersecting XY at A $\angle$X'Y' at B. Prove that $\angle$AOB = 90°.

Sol. Join OC. In ΔAPO and ΔACO, we have

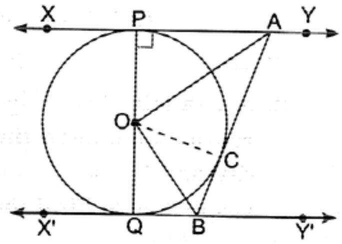
AP = AC [Tangents drawn from external point A]

AO = OA [Common]

PO = OC [Radii of the same circle]

$\therefore$ $\Delta APO \cong \Delta ACO$ (By SSS criterion of congruence)

$\therefore$ $\angle PAO = \angle CAO$ (CPCT)



$\Rightarrow$ $\angle PAC = 2\angle CAO$

Similarly, we can prove that

$\Delta OQB \cong \Delta OCB$

$\therefore$ $\angle QBO = \angle CBO$ $\Rightarrow$ $\angle CBQ = 2\angle CBO$

Now, $\angle$PAc + $\angle$CBQ = 180° [Sum of interior angles on the same side of transversal is 180°]

$\Rightarrow$ 2∠CAO + 2∠CBO = 180°

$\Rightarrow$ ∠CAO + ∠CBO = 90°

$\Rightarrow$ 180° – ∠AOB = 90° [∵ ∠CAO + ∠CBO + ∠AOB = 180°]

$\Rightarrow$ 180° – 90° = ∠AOB $\Rightarrow$ ∠AOB = 90°

CHAPTER - 8 : TRIGONOMETRY

Q59. If $\sec\theta = x + \frac{1}{4x}$, prove that sec θ + tan θ = 2x or $\frac{1}{2x}$

Sol. Let $\sec\theta = \tan\theta = \lambda$ ----(i)

We know that, $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1 \quad \Rightarrow \quad \lambda(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{\lambda}$ ---(ii)

Adding equations (i) and (ii), we get

$2\sec\theta = \lambda + \frac{1}{\lambda} \quad \Rightarrow \quad 2\left(x + \frac{1}{\lambda}\right) = \lambda + \frac{1}{\lambda}$

$\Rightarrow \quad 2x + \frac{1}{2x} = \lambda + \frac{1}{\lambda}$

om comparing, we get $\lambda = 2\text{x}$ or $\lambda = \frac{1}{2x}$

$\Rightarrow \quad \sec\theta + \tan\theta = 2\text{x or } \frac{1}{2\text{x}}$

Q60. Find an acute angle θ, when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Sol. We have

$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \quad \Rightarrow \quad \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

[Dividing numerator & denominator of the LHS by $\cos\theta$]

$$\Rightarrow \quad \frac{1-\tan\theta}{1+\tan\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$$

On comparing we get

$$\Rightarrow \quad \tan\theta=\sqrt{3} \quad \Rightarrow \quad \tan\theta=\tan 60^0 \quad \Rightarrow \quad \theta = 60°$$

Q61. If $\cos ec\theta=\frac{13}{12}$**, evaluate** $\frac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta}$

Sol. Given $\cos ec\theta=\frac{13}{12}$, then $\sin\theta=\frac{12}{13}$

$$\cos^2\theta=1-\sin^2\theta=1-\left(\frac{12}{13}\right)^2=\frac{169-144}{169}=\frac{25}{169}$$

$$\cos\theta=\frac{5}{13}$$

Now, $\frac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta}=\frac{2\text{ x }\frac{12}{13}-3\text{ x }\frac{5}{13}}{4\text{ x }\frac{12}{13}-9\text{ x }\frac{5}{13}}=\frac{24-15}{48-45}=\frac{9}{3}=3$

Q62. Prove that $\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}=2\text{ cosec A}$

Sol. LHS $\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}$

$$=\frac{\tan A(1-\sec A)-\tan A(1+\sec A)}{(1+\sec A)(1-\sec A)}$$

$$=\frac{\tan A-\tan A\sec A-\tan A-\tan A\sec A}{1-\sec^2 A}$$

$$=\frac{-2\tan A\sec A}{1-\sec^2 A}=\frac{2\tan A\sec A}{\sec^2 A-1}$$

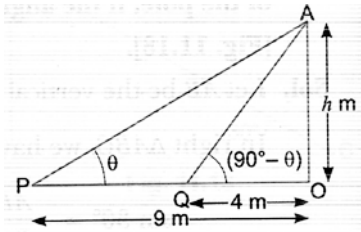
$$=\frac{2\tan A\sec A}{\tan^2 A} \qquad (\because \sec^2 A = 1+\tan^2 A)$$

$$=\frac{2\sec A}{\tan A}=\frac{\frac{2}{\cos A}}{\frac{\sin A}{\cos A}}=\frac{2}{\sin A}=2\text{ cosec } A = \text{RHS}$$

CHAPTER - 9 : HEIGHT AND DISTANCE

Q63. The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Sol. Let OA be the tower of height h metre and P, Q be the two points at distance of 9m and 4m respectively from the base of the tower.



Now, we have OP = 9 m, OQ = 4 m,

Let $\angle APO = \theta, \angle AQO = (90^0 - \theta)$

and OA = h metre (Fig)

Now, in Δ POA, we have

$$\tan\theta = \frac{OA}{OP} = \frac{h}{9} \quad \Rightarrow \tan\theta = \frac{h}{9} \qquad \text{---(i)}$$

Again, in Δ AQO, we have

$$tan\ (90^0 - \theta) = \frac{OA}{OQ} = \frac{h}{4} \quad \Rightarrow cot\theta = \frac{h}{4} \qquad \text{---(ii)}$$

Multiplying (i) and (ii), we have

$$\tan\theta \text{ x } \cot\theta = \frac{\text{h}}{9} \text{ x } \frac{\text{h}}{4} \quad \Rightarrow 1 = \frac{h^2}{36} \quad \Rightarrow h^2 = 36$$

$$h = \pm 6$$

Height cannot be negative

Hence, the height of the tower is 6 metre.

Q64. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

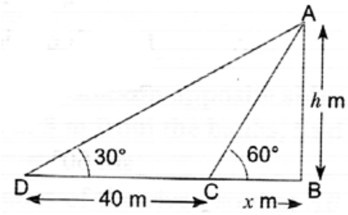
Sol. In fig. AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m.

According to the question, DB is 40 m longer than BC.

So, BD = (40 + x)m

Now, we have two right triangles ABC and ABD



In ΔABC, $\tan 60^0 = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow$ $\sqrt{3} = h$ ---(i)

In ΔABD, $\tan 30^0 = \frac{AB}{BD}$

i.e. $\frac{1}{\sqrt{3}} = \frac{4}{x+40}$ ---(ii)

Using (i) in (ii), we get $(x\sqrt{3})\sqrt{3} = \text{x} + 40, \text{i.e.}\, 3\,\text{x} = \text{x} + 40$

i.e., x = 20 [From (i)]

So, $h = 20\sqrt{3}$

Therefore, the height of the tower is $20\sqrt{3}$ m.

Q65. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 3m from the banks, find the width of the river.

Sol. In Fig. A and B represent points on the bank on opposite side of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3m, i.e., DP = 3m. We are interested to determine the width of the river, which is the length of the side AB of the ΔAPB.

In the right ΔADP, $\angle$A = 30°

So, $\tan 30^0 = \frac{PD}{AD}$

i.e., $\frac{1}{\sqrt{3}} = \frac{3}{AD}$ or $AD = 3\sqrt{3}$ m

Also, in right ΔPDB,

$\frac{PD}{AD} = \tan 45^0$ $\Rightarrow$ $\frac{3}{DB} = 1$

$\therefore$ DB = 3m

Now, AB = BD + AD = 3 + $3\sqrt{3}$ = 3 (1+$\sqrt{3}$) m

Therefore, the width of the river is $3(\sqrt{3}+1)$ m