

DELHI PUBLIC SCHOOL, JAMMU

ASSIGNMENT FOR CLASS 12TH

SUBJECT:- MATHS

SESSION:- 2018-19

CHAPTERS :

- RELATIONS AND FUNCTIONS
- INVERSE TRIGONOMETRIC FUNCTIONS
- DETERMINANTS
- MATRICES
- CONTINUITY AND DIFFERENTIABILITY
- APPLICATION OF DERIVATIVES
- INTEGRALS

1. RELATIONS AND FUNCTIONS:-

Q1. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $F = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

Q2. If $A = \{1, 2, 3, \dots, 9\}$ and R is a relation in AXA defined by $(a, b), (c, d)$ in AXA, if $a + d = b + c$ for $(a, b), (c, d)$ in AXA. Prove that R is an equivalence relation class $[(2, 5)]$.

Q3. If $f, g : \mathbb{R} \rightarrow \mathbb{R}$ are two functions defined as $f(x) = |x|^{-x}$ and $g(x) = |x|^{+x}$, for all $x \in \mathbb{R}$. Then find fog and gof.

Q4. Let $F : \mathbb{N} \rightarrow \mathbb{N}$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $F : \mathbb{N} \rightarrow S$, where S is the range of F, is invertible.. Find the inverse of F and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

Q5. If N denotes the set of all natural numbers and R be the relation on NXN defined by $(a, b)R(c, d)$, if $ad(b+c) = bc(a+d)$. show that R is an equivalence relation.

Q6. Let $A = \{a, b, c, d\}$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b+ad)$ for $(a, b), (c, d) \in A$. Determine whether $*$ is commutative and associative. Then w.r.t to $*$ on A . Find the identity element in A , also find the invertible elements of A .

2. INVERSE TRIGONOMETRIC FUNCTIONS:-

Q1. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$; $xy < 1$, then write the value of $x+y+xy$.

Q2. Write the value of $\cos^{-1} \left(\frac{-1}{2}\right) + 2 \sin^{-1} \frac{1}{2}$

Q3. Solve the equation for x : $\cos(\tan^{-1} x) = \sin(\cot^{-1} \frac{3}{4})$

Q4. If $\tan^{-1} \left(\frac{1}{1+1.2}\right) + \tan^{-1} \left(\frac{1}{1+2.3}\right) + \dots + \tan^{-1} \left(\frac{1}{1+n(n+1)}\right) = \tan^{-1} \theta$

Q5. Prove that: $\cos^{-1} x + \cos^{-1} \left(\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right) = \frac{\pi}{3}$.

3. MATRICES AND DETERMINANTS:-

Q1. Write all the number of possible matrices of order 2×2 with entry 1, 2 or 3

Q2. If matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = KA$, then write the value of K .

Q3. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$, Find the value of k .

Q4. Write 2×2 matrix which is both symmetric and skew-symmetric.

Q5. Show that all the diagonal elements of a skew-symmetric matrix are zero.

Q6. Express the matrix as a sum of symmetric and skew-symmetric matrices and verify the result

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Q7. Using elementary transformation, find the inverse of $I) \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

Q8. A trust fund has Rs. 35000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an NGO. Using matrix method, determine how to divide Rs. 35000 among two type of bonds, an annual interest is Rs. 3200.

Q9. Find the maximum value of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin\theta & 1 \\ 1 & 1 & 1 + \cos\theta \end{bmatrix}$

Q10. Find the equation of the line joining A(1,3) and B(0,0), using determinants and find the value of k if D(k,0) is a point such that area of a triangle ABD is 3 square meters.

Q11. Let A be a square matrix of order 3x3. write the value of $|2A|$, where $|A| = 4$

Q12. Using properties of determinants, prove that:

a) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c$

b) $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x+y+z)$

c) $\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$, show that ΔABC is isosceles.

$$d) \begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 + a^2 - b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$

Q13. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the equations: $x-y+2z=1, 2y-3z=1$ and $3x-2y+4z=2$.

4. CONTINUITY AND DIFFERENTIABILITY:-

Q1. Determine the value of K for which the following function is continuous at $x=3$:

$$f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases} \text{ is continuous at } x=0.$$

$$Q2. \text{ If } f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin x}{x}, & \text{if } x < 0 \\ p, & \text{if } x = 0 \\ \frac{\sqrt{1+bx-1}}{x}, & \text{if } x > 0 \end{cases} \text{ is continuous at } x=0, \text{ then find the values of } a \text{ and } b.$$

Q3. Differentiate $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ w.r.t $\sin^{-1} \frac{2x}{1+x^2}$, when $x \neq 0$.

Q4. If $y = P e^{ax} + Q e^{bx}$, then show that $\frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = 0$

Q5. Differentiate w.r.t x : $x^{\cos x} + \sin x^{\cos x}$

5. APPLICATION OF DERIVATIVES:-

Q1. The sides of an equilateral triangle are increasing at the rate of 2cm per second. Find the rate at which the area is increasing, when side is 10cm?

Q2. Using differentials, Find the approximate value of $(3.968)^{\frac{3}{2}}$

Q3. Find the interval in which the function $f(x) = x^4 - 4x^3 - 12x^2 + 5$ is i) strictly increasing, ii) strictly decreasing'.

Q4. Find the point on the curve $y = x^3 - 11x + 5$ at which equation of tangent is $y = x - 11$

Q5. A window is in the form of rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Q6. Show that the semi vertical angle of the cone of the maximum volume and of given slant height is

$$\cos^{-1} \frac{1}{\sqrt{3}}$$

6. INTEGRALS:-

indefinite integral:

Q1. Integrate the following functions:

a) Find $\int \frac{2x}{(x^2+1)(x^2+2)^2} dx$ b) $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)} dx$ c) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$

d) $\int (x+3)\sqrt{3-4x-x^2} dx$ e) $\int \frac{(x^2+1)e^x}{(x+1)^2} dx$ f) $\int \frac{x^2+4}{x^4+16} dx$

Q2. DEFINITE INTEGRAL:

EVALUATE: a) $\int_1^4 (|x-1| + |x-2| + |x-4|) dx$ b) $\int_0^1 \frac{\log|1+x|}{1+x^2} dx$

c) $\int_1^3 (e^x + x^2 + 1) dx$ as a limit of a sum d) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$

6. APPLICATION OF INTEGRALS:

Q1. Using integration, find the area of region bounded by the triangle whose vertices are (-2,0)(0,4)(2,3)

Q2. Using integration, Find the area of the region bounded by the lines $2x+y=4, 3x-2y=6$ and $x-3y+5=0$