

Delhi Public School, Jammu
Question Bank for Half-Yearly Exam (2018-19)

Subject: Mathematics

Class: 9th

Lines and Angles

1. Prove that if two lines intersect, then the vertically opposite angles are equal.

Sol: Let lines AB and CD intersect at O.

We have to prove: $\angle AOD = \angle BOC$ and $\angle AOC = \angle BOD$

Proof:

Since, ray OA stands on line CD.

$$\Rightarrow \angle AOC + \angle AOD = 180^\circ \quad \text{-----1.}$$

Again since, ray OC stands on line AB

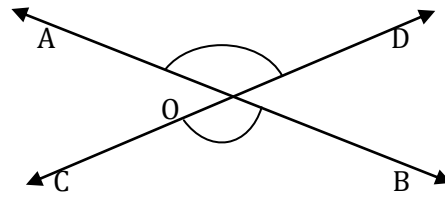
$$\Rightarrow \angle AOC + \angle BOC = 180^\circ \quad \text{-----2.}$$

From 1st and 2nd, we have

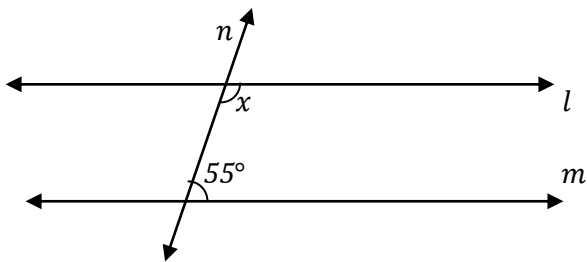
$$\angle AOC + \angle AOD = \angle AOC + \angle BOC$$

$$\angle AOD = \angle BOC$$

Similarly, we can prove that $\angle AOC = \angle BOD$



2. Find the value of x for which the line *l* and *m* are parallel.



Sol: Two lines are parallel when angles on the same side of transversal are supplementary.

$$x + 55^\circ = 180^\circ$$

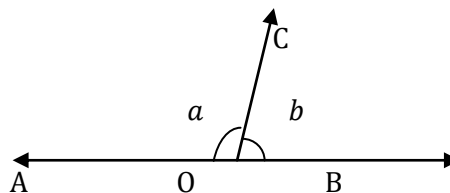
$$\Rightarrow x = 180^\circ - 55^\circ = 125^\circ$$

3. In $\angle AOC$ and $\angle BOC$ form a linear pair. If $a - b = 20^\circ$, find the value of a and b.

$$\text{Sol: } a + b = 180^\circ \quad (\text{linear pair}) \quad \text{---1.}$$

$$a - b = 20^\circ \quad (\text{given}) \quad \text{---2.}$$

Adding 1st and 2nd, we get



$$2a = a = 100^\circ \quad \text{or } a = 100^\circ$$

$$\text{And } b = 80^\circ$$

4. If $x + y = w + z$, then prove that AOB is a line.

Sol: As sum of all angles about a point is 360°

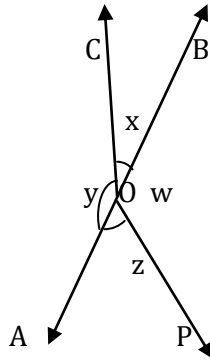
$$\text{Therefore, } x + y + w + z = 360^\circ$$

$$(x + y) + (w + z) = 360^\circ$$

$$\text{As } x + y = w + z$$

$$\text{So, } 2x + 2y = 360^\circ \quad \text{or } x + y = 180^\circ$$

This shows that AOB is a straight line.



5. In a fig, $AB \parallel CD$. Determine x.

Sol: Through O, draw a line l parallel to both AB and CD.

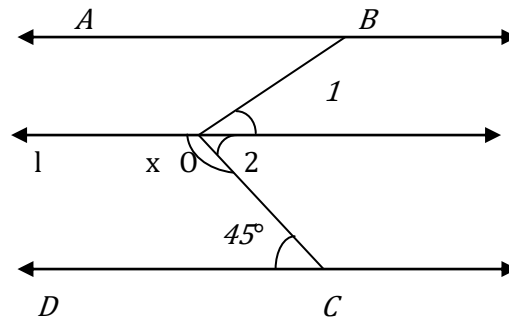
$$\angle 1 = \angle AOB = 30^\circ \quad (\text{Alternate interior angles})$$

$$\angle 2 = \angle DCO = 45^\circ \quad (\text{Alternate interior angles})$$

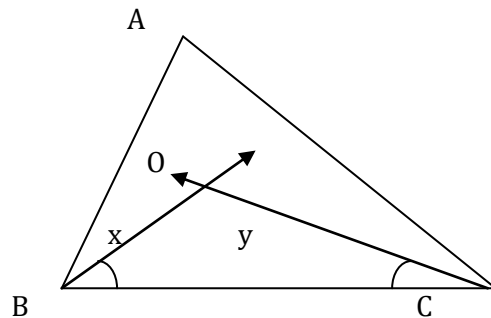
$$\angle BOC = \angle 1 + \angle 2$$

$$\angle BOC = 30 + 40^\circ = 75^\circ$$

$$\text{So, } \angle x = 360^\circ - 75^\circ = 285^\circ$$



6. If in $\triangle ABC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Prove that $\angle BOC = 90^\circ + \frac{1}{2} \angle A$.



sol: Let $\angle B = 2x$ and $\angle C = 2y$

\therefore OB and OC bisect $\angle B$ and $\angle C$ respectively.

$$\therefore \angle BOC = \frac{1}{2} \angle B = \frac{1}{2} \times 2x = x$$

$$\text{and } \angle OCB = \frac{1}{2} \angle C = \frac{1}{2} \times 2y = y$$

Now, in $\triangle BOC$, we have

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

$$\angle BOC + x + y = 180^\circ$$

$$\text{or } \angle BOC = 180^\circ - (x + y) \quad \text{-----1.}$$

Now in $\triangle ABC$, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 2x + 2y = 180^\circ$$

$$\text{or } 2(x + y) = 180^\circ - \angle A$$

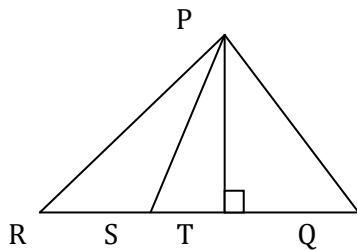
$$x + y = \frac{1}{2}(180^\circ - \angle A) \quad \text{-----2.}$$

From 1st and 2nd, we have

$$\angle BOC = 180^\circ - (x + y) = 180^\circ - \frac{1}{2}(180^\circ - \angle A)$$

$$\angle BOC = 90^\circ + \frac{1}{2}\angle A$$

7. In the figure, PS is the bisector of $\angle QPR$; $PT \perp RQ$ and $Q > R$. Show that $\angle TPS = \frac{1}{2}(\angle Q - \angle R)$.



sol: Since PS is the bisector of $\angle QPR$

$$\therefore \angle QPS = \angle RPS = x \text{ (say)}$$

In $\triangle PRT$, we have

$$\angle PRT + \angle PTR + \angle RPT = 180^\circ$$

$$\angle PRT + 90^\circ + \angle RPT = 180^\circ$$

$$\angle PRT + \angle RPS + \angle TPS = 90^\circ$$

$$\angle PRT + x + \angle TPS = 90^\circ \quad (\angle RPS = x)$$

$$\angle PRT = \angle R = 90^\circ - \angle TPS - x \quad \text{-----1.}$$

In $\triangle PQT$, we have

$$\angle PQT + \angle PTQ + \angle QPT = 180^\circ$$

$$\angle PQT + 90^\circ + \angle QPT = 180^\circ$$

$$\angle PQT + \angle QPS - \angle TPS = 90^\circ$$

$$\angle PQT + x - \angle TPS = 90^\circ \quad (\angle QPS = x)$$

$$\angle PQT = \angle Q = 90^\circ + \angle TPS - x \quad \text{----- 2.}$$

Subtracting 1st from 2nd, we have

$$\angle Q - \angle R = (90^\circ + \angle TPS - x) - (90^\circ - \angle TPS - x) = 2\angle TPS$$

$$2\angle TPS = \angle Q - \angle R$$

$$\angle TPS = \frac{1}{2}(\angle Q - \angle R)$$

EUCLID'S GEOMETRY

Q1. If a point 'C' lies between two points 'A' and 'B' such that $AC=BC$ then prove that $AC=\frac{1}{2}AB$. Explain by drawing fig.

Sol: Given: $AC=BC$

To Prove: $AC=\frac{1}{2}AB$

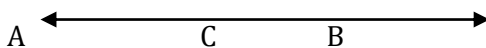
Proof:- $AC=BC$

Adding AC to both sides

$$AC+AC=BC+AC$$

$$2AC=AB$$

$$AC=\frac{1}{2}AB$$



Q2. In a fig . If $AC=BD$ then prove that $AB=CD$.

Sol: Given $AC=BD$

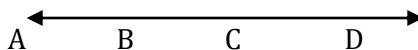
To Prove $AB=CD$

Proof: $AC=BD$

Subtracting BC on both sides, we get

$$AC-BC=BD-BC \text{ (by Euclid's Axiom 3)}$$

$$\text{Therefore } AB=CD$$



Q3. Why is Axiom 5, in the list of Euclid's Axiom considered a 'universal truth'?

Ans: We know that whole number is always greater than a part.

Q4. Thales belong to the country

a) babylonia b) Egypt c) Greece d) Rome

Ans : Greece

Q5: A pyramid is a solid figure, the base of which is

a) only a triangle b) only a Square

c) only a rectangle c) Any polygon

Ans: only a rectangle

Real Numbers

1. If $a = 3 + 2\sqrt{2}$, then find the value of $a^2 + \frac{1}{a^2}$

sol: $a = 3 + 2\sqrt{2}$ and $\frac{1}{a} = \frac{1}{3+2\sqrt{2}}$

$$\frac{1}{a} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$$

$$a + \frac{1}{a} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

Putting the value of $a + \frac{1}{a}$, we get

$$6^2 = a^2 + \frac{1}{a^2} + 2$$

$$a^2 + \frac{1}{a^2} = 36 - 2 = 34$$

2. Simplify: $\left[9(64^{1/3} + 125^{1/3})^3\right]^{1/4}$

sol: $\left[9(64^{1/3} + 125^{1/3})^3\right]^{1/4}$

$$= \left[9((4^3)^{1/3} + (5^3)^{1/3})^3\right]^{1/4}$$

$$= [9(4 + 5)^3]^{1/4}$$

$$= (9 \times 9^3)^{1/4} = (9^4)^{1/4} = 9$$

3. Simplify: $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

sol:

$$\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$$

$$= (3^4)^{1/4} - 8(6^3)^{1/3} + 15(2^5)^{1/5} + (15^2)^{1/2}$$

$$= 3 - 48 + 30 + 15 = 48 - 48 = 0$$

4. Express $0.\bar{6}$ in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

sol:

Let $x = 0.\bar{6}$, then $x = 0.666\dots$

$10x = 6.666\dots$

subtract 1st from 2nd, we get

$$9x = 6$$

$$\Rightarrow x = 6/9$$

$$x = 2/3$$

5. Find three rational number $\frac{5}{7}$ and $\frac{6}{7}$.

sol:

we have $\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}$ and $\frac{6}{7} = \frac{6 \times 4}{7 \times 4} = \frac{24}{28}$

therefore, three rational number between $\frac{5}{7}$ and $\frac{6}{7}$ are $\frac{21}{28}$, $\frac{22}{28}$ and $\frac{23}{28}$

6. Rationalise; $\frac{1}{7+5\sqrt{2}}$

sol: $\frac{1}{7+5\sqrt{2}}$

$$= \frac{1}{7+5\sqrt{2}} \times \frac{7-5\sqrt{2}}{7-5\sqrt{2}}$$

$$= \frac{7-5\sqrt{2}}{7^2 - (5\sqrt{2})^2}$$

$$= \frac{7-5\sqrt{2}}{49-50} = \frac{7-5\sqrt{2}}{-1}$$

$$= -7 + 5\sqrt{2}$$

7. Simplify; $\sqrt[12]{(x^4)^{1/3}}$

sol:

$$\sqrt[12]{(x^4)^{1/3}}$$

$$= [(x^4)^{1/3}]^{1/12}$$

$$= (x)^{4 \times \frac{1}{3} \times \frac{1}{12}} = x^{1/9}$$

8. Is the number $(3 - \sqrt{7})(3 + \sqrt{7})$ rational or irrational?

sol:

Given number is;

$$(3 - \sqrt{7})(3 + \sqrt{7})$$

$$= 3^2 - (\sqrt{7})^2$$

$$= 9 - 7 = 2, \text{ which is rational.}$$

Heron's Formula

1. Find the areas of trapezium whose parallel sides are 25cm and 13cm long and distance between them is 8cm.

sol: Area of trapezium = $\frac{1}{2}(\text{sum of parallel side}) \times \text{height}$

$$= \frac{1}{2} \times (25 + 13) \times 8$$

$$= 152 \text{ cm}^2$$

2. If the area of equilateral triangle is $16\sqrt{3} \text{ cm}^2$, then find the perimeter of triangle.

sol:

$$\text{Area of equilateral triangle} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} a^2 = 16\sqrt{3}$$

$$\Rightarrow a^2 = 64 \text{ or } a = 8 \text{ cm}$$

$$\text{Perimeter of the equilateral triangle} = 3a = 3 \times 8 = 24 \text{ cm}$$

3. The perimeter of isosceles triangle is 32cm. the ratio of equal side to its base is 3: 2. Find the area of triangle.

sol:

Let each side of isosceles triangle equal to 3x cm.

base of isosceles triangle equal to 2x cm.

$$\text{perimeter} = 3x + 3x + 2x = 32$$

$$8x = 32 \text{ or } x = 4 \text{ cm}$$

sides are 3x=12cm, 3x=12cm and 2x=8cm

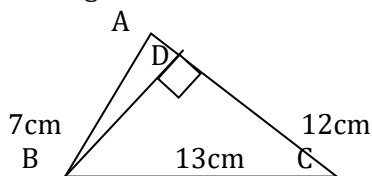
$$s = \frac{a+b+c}{2} = \frac{12+12+8}{2} = 16 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-12)(16-12)(16-8)}$$

$$= \sqrt{16 \times 4 \times 4 \times 8} = 32\sqrt{2} \text{ cm}^2$$

4. The length of side of triangle are 7cm, 13cm and 12cm. Find the length of perpendicular from the opposite vertex to the side whose length is 12cm.



sol:

$$s = \frac{a+b+c}{2} = \frac{7+13+12}{2} = 16 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-7)(16-13)(16-12)} = \sqrt{16 \times 9 \times 3 \times 4} = 24\sqrt{3} \text{ cm}^2$$

$$\text{also, area of triangle ABC} = \frac{1}{2} \times AC \times BD$$

$$24\sqrt{3} = \frac{1}{2} \times 12 \times BD$$

$$BD = 4\sqrt{3} \text{ cm}$$

5. If each side of triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle.

sol: Let a, b, c be the sides of the triangle.

$$s = \frac{a+b+c}{2} \quad \text{_____} \quad 1.$$

$$\text{Area of triangle, } \Delta = \sqrt{s(s-a)(s-b)(s-c)} = A$$

if each side is doubled, then

$$s' = \frac{2a+2b+2c}{2} = a + b + c \text{ -----}2.$$

from 1st and 2nd we get,

$$s' = 2s$$

$$\text{Area of new triangle, } \Delta' = A' = \sqrt{s'(s' - 2a)(s' - 2b)(s' - 2c)} = \sqrt{2s(2s - 2a)(2s - 2b)(2s - 2c)}$$

$$= \sqrt{16s(s - a)(s - b)(s - c)} = 4\sqrt{s(s - a)(s - b)(s - c)} = 4A$$

$$A' = 4A = 4:1$$

Topic-polynomial

Q1. If $5x - 4y = 15$ and $xy = 3$ find the value of $25x^2 + 16y^2$.

Sol. Squaring both sides

$$(5x)^2 + (4y)^2 - 2 \times 5x \times 4y = 225$$

$$25x^2 + 16y^2 - 40xy = 225$$

$$25x^2 + 16y^2 - 40 \times 3 = 225$$

$$25x^2 + 16y^2 = 225 + 120 = 345$$

Q2 If both $(x-2)$ and $(x-1/2)$ are factors of $px^2 + 5x + r$, show that $p=r$.

Sol. Let $f(x) = px^2 + 5x + r$, As $x-2$ is the factor of $f(x)$, so

$$f(2) = 0$$

$$px^2 + 5x + r = 0$$

$$4p + 10 + r = 0 \text{(1)}$$

Also $(x-1/2)$ is a factor of $f(x)$, so $f(1/2) = 0$

$$P(1/2)^2 + 5(1/2) + r = 0$$

$$P/4 + 5/2 + R = 0$$

$$P + 10 + 4r = 0 \text{(2)}$$

From 1&2

$$4p + 10 + r = p + 10 + 4r$$

$$4p - p = 10 + 4r - 10 - r$$

$$3p = 3r$$

$$P = r$$

Q3. If $a - b = 12$ & $ab = 14$ find $a^2 + ab + b^2$

Sol. $(a-b)^3 = 12^3$ cubing both side

$$a^3 - b^3 - 3ab(a-b) = 1728$$

$$a^3 - b^3 - 3 \times 14 \times 12 = 1728$$

$$a^3 - b^3 = 1728 + 504 = 2232$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) = 3 \times (40 + 31/2) = 333/2.$$

Q4. If $a^2 + b^2 + c^2 = 20$ & $ab + bc + ca = 8$ find $a + b + c$

Sol. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 20 + 2(ab + bc + ca) = 20 + 2 \times 8 = 36$

$$a + b + c = \pm 6$$

Q5. Without finding the cubes, factorise $(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$.

Sol. Let $a = x - 2y$, $b = 2y - 3z$, $c = 3z - x$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$$(x-2y)^3 + (2y-3z)^3 + (3z-x)^3 = 3(x-2y)(2y-3z)(3z-x).$$

Q6: If the polynomial $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leaves the same remainder when divided by $z-3$, find a .

Sol. Let $p(z) = az^3 + 4z^2 + 3z - 4$

$$P(3) = a \times 3^3 + 4 \times 3^2 + 3 \times 3 - 4 = 27a + 36 + 9 - 4 = 27a + 41 \text{(1)}$$

$$q(z) = z^3 - 4z + a$$

$$q(3) = 3^3 - 4 \times 3 + a = 27 - 12 + a = 15 + a \text{(2)}$$

$$\text{ATQ } p(3) = q(3)$$

$$27a + 41 = 15 + a$$

$$27a - a = -41 + 15 \text{ or } 26a = -26 \text{ or } a = -1.$$

Q7: Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$.

Sol: $x^3 - 8y^3 - 36xy - 216$

$$= x^3 + (-2y)^3 + (-6)^3 - 3(x)(-2y)(-6)$$

$$= (x-2y-6)(x^2+4y^2+36+2xy-12y+6x)$$

$$0(x^2+4y^2+36+2xy-12y+6x)$$

0

Q8: For what value of K, $(x+1)$ is a factor of $p(x)=kx^2-x-4$

Sol:

As $x+1$ is a factor of $p(x)$, so $p(-1)=0$

$$K(-1)^2 - (-1) - 4 = 0$$

$$K+1-4=0$$

$$k-3=0 \text{ or } k=3.$$

Q9: Evaluate $185 \times 185 - 15 \times 15$

$$\text{Sol: } (185)^2 - (15)^2$$

$$= (185+15)(185-15)$$

$$= 200 \times 170 = 34000.$$

Q10: Factorise; $2x^2-7x-15$

$$\text{Sol: } 2x^2-7x-15$$

$$= 2x^2-10x+3x-15$$

$$= 2x(x-5)+3(x-5)$$

$$= (x-5)(2x+3).$$

Q11. Find if $(-2x-5)$ is a factor of the polynomial $p(x) = 3x^4 + 5x^3 - 2x^2 - 4$ or not.

$$\text{Sol. As } p(x) = 3x^4 + 5x^3 - 2x^2 - 4$$

Put

$$\text{Now, } p\left(\frac{-5}{2}\right) = 3\left(\frac{-5}{2}\right)^4 + 5\left(\frac{-5}{2}\right)^3 - 2\left(\frac{-5}{2}\right)^2 - 4$$

$$= 3\left(\frac{625}{16}\right) + 5\left(\frac{-125}{8}\right) - 2\left(\frac{25}{4}\right) - 4$$

$$= \frac{1875}{16} - \frac{625}{8} - \frac{25}{2} - 4$$

$$= \frac{1875 - 1250 - 200 - 64}{16}$$

$$= \frac{361}{16} \neq 0$$

Q12. Factorise $x^2 + 3\sqrt{3}x + 6$

$$\text{Sol. } x^2 + 3\sqrt{3}x + 6 = x^2 + 2\sqrt{3}x + \sqrt{3}x + 6$$

$$= x(x + 2\sqrt{3}) + \sqrt{3}(x + 2\sqrt{3})$$

$$= (x + 2\sqrt{3})(x + \sqrt{3})$$

Q13. Simplify : $(5a+3b)^3 - (5a-3b)^3$

$$\text{Sol. } (5a+3b)^3 - (5a-3b)^3$$

$$= [(5a+3b) - (5a-3b)] [(5a+3b)^2 + (5a+3b)(5a-3b) + (5a-3b)^2]$$

$$= (5a+3b-5a+3b) \{ [25a^2 + 2 \times 5a \times 3b + 9b^2] + [(5a)^2 - (3b)^2] + [25a^2 - 2 \times 5a \times 3b + 9b^2] \}$$

$$= (6b) [25a^2 + 30ab + 9b^2 + 25a^2 - 9b^2 + 25a^2 - 30ab + 9b^2]$$

$$= 6b [75a^2 + 9b^2] = 18b[25a^2 + 3b^2]$$

Q14. If $x + \frac{1}{x} = 7$, then find the value of $x^3 + \frac{1}{x^3}$

$$\text{Sol. We have } x + \frac{1}{x} = 7$$

Cubing both side, we have

$$\left(x + \frac{1}{x}\right)^3 = 7^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 7 = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21 = 322$$

Q15 if $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

$$\text{Sol. As } x + y + z = 0 \quad (\text{given})$$

$$\Rightarrow x + y = -z \quad \dots (i)$$

Cubing both sides, we have

$$(x + y)^3 = (-z)^3$$

$$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$$

$$\Rightarrow x^3 + y^3 + z^3 + 3xy(x + y) = 0$$

$$\text{But } x + y = -z \quad [\text{From (i)}]$$

$$\Rightarrow x^3 + y^3 + z^3 + 3xy(-z) = 0$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz$$

Hence, proved.

Q16. if $x + 2y = 8$ and $xy = 8$, find the value of $x^3 + 8y^3$.

Sol. We have $x + 2y = 8$

Cubing both side, we have

$$(x + 2y)^3 = 8^3$$

$$x^3 + (2y)^3 + 3 \times x \times 2y(x + 2y) = 512$$

$$x^3 + 8y^3 + 6xy(x + 2y) = 512$$

$$x^3 + 8y^3 + 6 \times 8 \times 8 = 512$$

$$x^3 + 8y^3 + 384 = 512$$

$$x^3 + 8y^3 = 512 - 384 = 128$$

Q17. if $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a^3 + b^3$.

Sol. We have $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow 10^2 = 58 + 2ab$$

$$\Rightarrow 100 - 58 = 2ab$$

$$\Rightarrow 42 = 2ab$$

$$\Rightarrow ab = \frac{42}{2} = 21$$

$$\begin{aligned} \text{Now, } a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\ &= (a + b)(a^2 + b^2 - ab) \\ &= (10)(58 - 21) \\ &= 10 \times 37 = 370 \end{aligned}$$

Q18. Factorise : $6x^2 - 5x^2 - 13x + 12$

Sol. Consider $p(x) = 6x^2 - 5x^2 - 13x + 12$

By Hit and Trial Method, we find that

$$\begin{aligned} p(1) &= 6(1)^2 - 5(1)^2 - 13(1) + 12 \\ &= 6 - 5 - 13 + 12 \\ &= 18 - 18 \end{aligned}$$

$\therefore (x - 1)$ is a factor of $p(x)$

Now, divide $p(x)$ by $(x - 1)$, we have

$$\begin{array}{r} 6x^2 + x - 12 \\ x-1 \overline{) 6x^3 - 5x^2 - 13x + 12} \\ \underline{6x^3 - 6x^2} \\ (-) (+) \\ x^2 - 13x + 12 \\ \underline{x^2 - x} \\ (-) (+) \\ -12x + 12 \\ \underline{-12x + 12} \\ (+) (-) \\ \times \end{array}$$

Q19. Find what must be subtracted from the polynomial $4y^4 + 12y^3 + 6y^2 + 50y + 26$ so that the obtained polynomial is exactly divisible by $y^2 + 4y + 2$.

Sol. By using long division method , we have

$$\begin{array}{r}
 4y^2 - 4y + 14 \\
 y^2 + 4y + 2 \overline{) 4y^4 + 12y^3 + 6y^2 + 50y + 26} \\
 \underline{4y^4 + 16y^3 + 8y^2} \\
 (-) \quad (-) \quad (-) \\
 -4y^3 - 2y^2 + 50y \\
 \underline{-4y^3 - 16y^2 - 8y} \\
 (+) \quad (+) \quad (+) \\
 14y^2 + 58y + 26 \\
 \underline{14y^2 + 56y + 28} \\
 (-) \quad (-) \quad (-) \\
 2y - 2
 \end{array}$$

Q10. if $(x + 1)$ and $(x + 2)$ are the factors of $x^2 + 3x^2 - 3ax + \beta$, then find α and β .

Sol. Let $p(x) = x^2 + 3x^2 - 3ax + \beta$

Since $(x + 1)$ and $(x + 2)$ are factors of $p(x)$

\therefore Put $x = -1$ and -2 , we obtain

$$p(-1) = (-1)^3 + 3(-1)^3 - 3\alpha(-1) + \beta = 0$$

$$\Rightarrow -1 + 3 + 3\alpha + \beta = 0$$

$$\Rightarrow 3\alpha + \beta = -2 \quad \text{..(i)}$$

$$p(-2) = (-2)^3 + 3(-2)^2 - 3\alpha(-2) + \beta = 0$$

$$\Rightarrow -8 + 12 + 6\alpha + \beta = 0$$

$$\Rightarrow 6\alpha + \beta = -4 \quad \text{..(ii)}$$

Subtracting (i) from (ii), we have

$$3\alpha = -2 \Rightarrow \alpha = -\frac{2}{3}$$

$$\text{From (i), we have } 3 \times \left(-\frac{2}{3}\right) + \beta = -2 \Rightarrow \beta = 0$$

Hence, value of α and β are $\alpha = -\frac{2}{3}$ and $\beta = 0$.

Topic: Triangles

1. Show that in a right angled triangle, the hypotenuse is the longest side.

Sol. Let ABC be a right angled triangle in which $\angle B = 90^\circ$.

$$\text{But } \angle B + \angle C + \angle A = 180^\circ.$$

$$90^\circ + \angle C + \angle A = 180^\circ$$

$$\angle C + \angle A = 90^\circ$$

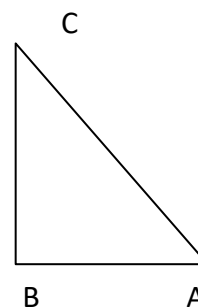
$\angle C$ and $\angle A$ are acute angles

$$\angle C < 90^\circ \text{ and } \angle A < 90^\circ$$

$$\angle C < \angle B \text{ and } \angle A < \angle B$$

$AB < AC$ and $BC < AC$ (Sides opposite to greater angle is larger)

In a right triangle, the hypotenuse is the longest side.



2. Prove that angles opposite to equal sides of an isosceles triangles are equal.

Sol. Given: In a triangle ABC, $AB=AC$

To Prove: $\angle B = \angle C$

Construction: Draw AD perpendicular to BC

Proof: In triangles ADC and ADB

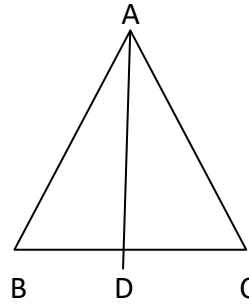
$AB = AC$ (given)

$\angle BAD = \angle CAD$

$AD=AD$ (common)

By SAS Criteria $\triangle ABD$ is congruent to $\triangle ACD$

Therefore $\angle B = \angle C$



3. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Sol. In triangles BCF AND CEF, we have

$\angle BFC = \angle CEB$ (each 90°)

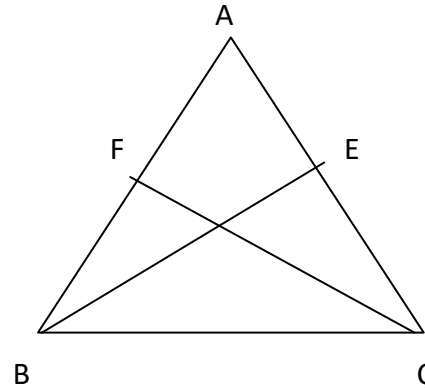
Hyp BC= Hyp BC (common)

$FC=EB$

By R.H.S Criteria of congruence, we have

$\triangle BCF \approx \triangle ECB$ (C.P.C.T)

Now in $\triangle ABC$, $\angle ABC = \angle ACB$



$AB=AC$ (Sides opposite to equal angles are equal).

Therefore, $\triangle ABC$ is an isosceles triangle.

4. In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is isosceles triangle in which $AB=AC$.

Sol. In $\triangle ABD$ AND $\triangle ACD$, we have

$DB= DC$ (given)

$\angle ADB = \angle ADC$ (Each 90°)

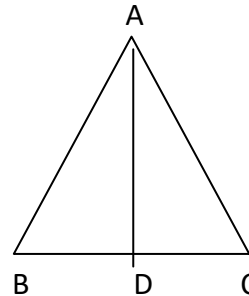
$AD=AD$

By SAS criteria of congruence, we have

$\triangle ABD \approx \triangle ACD$

$AB = AC$ (C.P.C.T)

Hence $\triangle ABC$ is isosceles.



5. Show that the angles of an equilateral triangle are 60° each.

Sol: let $\triangle ABC$ be an equilateral triangle so that $AB=AC=BC$.

$AB=AC$

$\angle B = \angle C$ (Angles opposite to equal sides are equal)(1)

Also, $CB=CA$

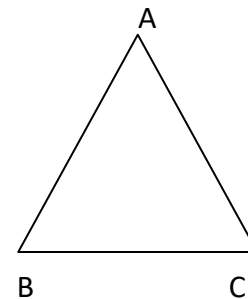
$\angle A = \angle B$ (Angles opposite to equal sides are equal).....(2)

From above equations , $\angle A = \angle B$

Also, $\angle A + \angle B + \angle C = 180^\circ$

$3\angle A = 180^\circ$

$\angle A = 60^\circ$



Coordinate Geometry

1. Find the ordinate in each of the following;

(a) $(-8, 6)$ (b) $(4, -11)$, (c) $(-5, -1)$

sol: (a) $=6$

(b) $=-11$

(c) $=-1$

2. In which quadrant does the following point lies;

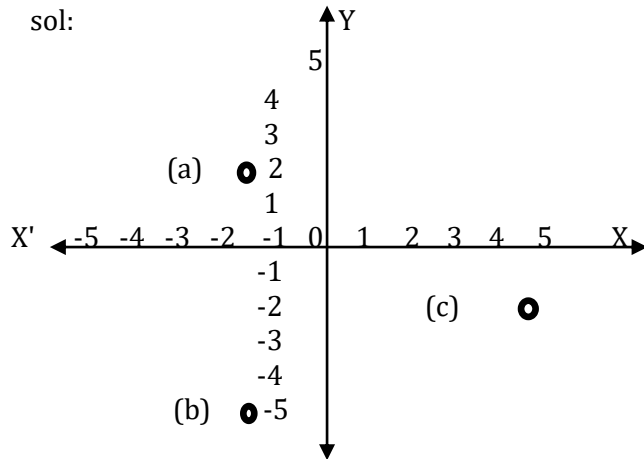
(a) $(4, 3)$, (b) $(-5, 6)$, (c) $(8, -4)$

sol: (a) First quadrant (b) second quadrant (c) Fourth quadrant

3. Plot the following points in Cartesian plane;

(a) $(2, -2)$, (b) $(-2, -5)$, (c) $(4, -2)$

sol:



4. Find the mirror images of the following points through y-axis;

(a) $(2, 3)$ and (b) $(-3, 5)$

sol: (a) $(-2, 3)$ and (b) $(3, 5)$

5. Plot the point $P(2, -3)$ on the graph and from it draw PM and PN as perpendicular to x-axis

and y-axis respectively. Give the coordinates of M and N .

sol:

