DELHI PUBLIC SCHOOL JAMMU
HALF YEARLY ASSIGNMENT
(SESSION 2019-20)

SUB: MATHEMATICS
CLASS: XII

1. valuate: $\int \frac{1}{x-\sqrt{x}} dx.$
2. Find: $\int e^x \frac{\sqrt{1+sin2x}}{1+cos2x} dx.$
3. Find: $\int \frac{e^x(x-3)}{(x-1)^2} dx.$
4. Find : $\int \frac{\cos\theta}{(4+sin^2\theta)(5-4cos^2\theta)}\ d\theta.$
5. Evaluate: $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-cos2x} dx.$
6. Evaluate: $\int_0^1 \frac{1}{e^x+e^{-x}} dx.$
7. Evaluate: $\int_{-1}^{\frac{3}{2}} |xsin(\pi x)| dx.$
8. Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{sinx}} dx$
9. Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx.
10. Evaluate: $\int \frac{cos2x}{(cosx+sinx)^2} dx.$
11. Evaluate:$\int \frac{logx}{x^2} dx.$
12. Evaluate: $\int \frac{sinx}{(cos^2x+1)(cos^2x+4)} dx.$
13. If $\int_0^a 3x^2\, dx = 8,$ find the value of a.
14. Evaluate: $\int \frac{1}{\sqrt{sin^3x \sin(x+\alpha)}} dx$
15. Evaluate $\int \frac{2+\sin x}{1+\cos x} e^{x/2}$ dx.
16. Show that $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2}\pi.$
17. Evaluate$\int \frac{x^2-1}{x^4+1} dx.$
18. Evaluate $\int_0^2 x - [x]\, dx.$
19. Evaluate $\int \frac{2+\sin x}{1+\cos x} e^{x/2}$ dx

13 .Evaluate: $\int_0^{\pi} \frac{xtanx}{secx+tanx} dx.$

20. Evaluate: $\int_0^{\frac{\pi}{4}} \left(\sqrt{tanx} + \sqrt{cotx}\right) dx.$

21. Show that the volume of the greatest cylinder that can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3\tan^2\alpha$.
22. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units . Show that the maximum volume of the box is $c^3/6\sqrt{3}$ cubic units .
23. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.
24. Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

25. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.
26. A window of perimeter (including the base of the arc) is in the form of a rectangle surrounded by a semi circle. The semi-circular portion is fitted with coloured glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. Show that the ratio of the length and breadth of the rectangle is 6:6+π, so that the window transmits maximum light.
27. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.
28. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
29. A jet of enemy is flying along the curve $y=x^2+2$ and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet.
30. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.
31. Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.
32. A tank with rectangular base and rectangular sides open at the top is to be constructed so that it's depth is 2m and volume is 8m3. If building of tank cost Rs. 70 per *sq* meter for the base and Rs. 45per *sq* meter for the sides. What is the cost of least expensive tank.
33. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.
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36. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units . Show that the maximum volume of the box is $c^3/6\sqrt{3}$ cubic units .
37. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.
38. Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

39. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.
40. A window of perimeter (including the base of the arc) is in the form of a rectangle surrounded by a semi circle. The semi-circular portion is fitted with coloured glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. Show that the ratio of the length and breadth of the rectangle is 6:6+π, so that the window transmits maximum light.
41. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.
42. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
43. If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.
44. A tank with rectangular base and rectangular sides open at the top is to be constructed so that it's depth is 2m and volume is 8m3. If building of tank cost Rs. 70 per *sq* meter for the base and Rs. 45per *sq* meter for the sides. What is the cost of least expensive tank.
45. A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when length of the field is twice its breadth.
46. A window of fixed perimeter (including the base of the triangle) is in the form of a rectangle surmounted by an equilateral triangle. The triangular portion is filled with colored glass while the rectangular part is filled with clear glass. The coloured glass stops 30% light fall on it while clear glass only 1%. What is the ratio of the sides of the rectangle so that the window transmits the maximum light? Use sunlight to save electricity and serve the nation directly comments on it.
47. Evaluate: $\int_0^{\frac{\pi}{2}} sin2x \log{(tanx)}\, dx$.
48. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cos^2x}{cos^2x+4sin^2x} dx$.
49. Using integration, find the area of the region bounded by curves: y=$\sqrt{5-x^2}$ and y=$|x-1|$.
50. Find the area of the region {(x,y): $0 \le y \le x^2+1$; $0 \le y \le x+1$; $0 \le x \le 2$}.
51. Using method of integration, find the area of triangle ABC, co-ordinates of whose vertices are A (4, 1), C (8, 4).
52. Find the area enclosed between the parabola $4y=3x^2$ and the straight line 3x-2y+12=0.
53. Find the general solution of the differential equation $\frac{dy}{dx}+1=e^{x+y}$.
54. Find the general solution of the differential equation $(\cos x)y_1 - \cos 2x = \cos 3x$.
55. Find the particular solution of the differential equation:
$\frac{dx}{dy}+x\cot y = 2y + y^2 \cot y, (y \neq 0)$ given that y (0) =$\frac{\pi}{2}$.
56. Solve the differential equation: $\tan y \frac{dy}{dx} = \cos(x+y)+\cos(x-y)$.
57. If y(x) is a solution of $\left(\frac{2+sinx}{1+y}\right)\frac{dy}{dx}$ = - cosx and y (0) = 1, then find the value of y at x=$\frac{\pi}{2}$.
58. Find the particular solution of the differential equation
$tanx\frac{dy}{dx} = 2xtanx + x^2 - y$; $(tanx \neq 0)$, given that y=0 when x = $\frac{\pi}{2}$.

59. Solve: $y + \frac{d}{dx}(xy) = x(sinx + logx)$.

60. Show that the differential equation (x-y) dy =(x+2y) dx is homogenous. Also, find the general solution of the given differential equation.

61. Find the particular solution of the differential equation y$e^y dx$=($y^3 + 2xe^y$)dy, y(0)=1.

62. Verify that $ax^2+by^2=1$ is a solution of the differential equation $x(yy_2+y_1^2) = yy_1$.

63. Find the differential equation representing the family of curves y=ae^{bx+5}, where a and b are arbitrary constants.

64. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} - y + \left(\frac{dy}{dx} + \frac{d^3y}{dx^3}\right)^{\frac{3}{2}}$.

65. If $x\frac{dy}{dx} = y(logy - logx)$, then the solution of the equation is: