DELHI PUBLIC SCHOOL,JAMMU
HALF YEARLY EXAMINATION
SESSION(2024-25) sample paper

CLASS: XII M.M:80
SUBJECT: APPLIED MATHEMATICS (241) TIME:3HRS

GENERAL INSTRUCTIONS:

- This question paper contains-**Five sections**A, B,C,D and E. Each Section is compulsory.However, there are internal choices in some questions.
- **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
- **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
- **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
- **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
- **Section E** has 3 **source based/ case based/passage based/ integrated unit of assessment** (4 marks each) with sub parts .

Section-A

(Multiple Choice Question)

Each question carries 1 mark

Q1. A boy's speed with the current is 15 km/h and the speed of the current is 2.5 km/h. The boy's speed against the current is

a. 8.5 km/h b. 9 km/h c. 10 km/h d. 12.5 km/h

Ans. c

Q2. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream it takes 4 hours. What is the speed of the boat in still water?

a. 4 km/h b. 9 km/h c. 8 km/h d. 10 km/h

Ans. b

Q3. $(18 \times 10) \pmod 7$ is

a. 5 b. 4 c. 3 d. 2

Ans. a

Q4. $[(3 \times 7) + 5] \pmod 4$ is

a. 5 b. 4 c. 3 d. 2

Ans. d

Q5. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+4 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, then x. y is equal to

a. 9 b. 14 c. 7 d. 17

Q6. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then x + y is equal to

a. 6 b. 14 c. 7 d. 18

Q7. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then (3A-B) is equal to

a. $\begin{bmatrix} 7 & 8 \\ 3 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 8 & 7 \\ 2 & 6 \end{bmatrix}$ c. $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$ d. $\begin{bmatrix} 7 & 8 \\ 2 & 6 \end{bmatrix}$

Q8. If $3A-B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then matrix A is

a. $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

Q9. The maximum value of $x^{1/x}$, $x > 0$ is

a. $e^{1/e}$ b. $\left(\frac{1}{e}\right)^e$ c. 1 d. none of these

Ans. b

Q10. If $ax + \frac{b}{x} \geq c$ for all positive x where a, b, > 0, then

a. $ab<\frac{c^2}{4}$ b. $ab>\frac{c^2}{4}$ c. $ab\geq \frac{c}{4}$ d. none of these

Ans. b

Q11. The minimum value of $\frac{x}{\log_e x}$ is

a. e b. 1/e c. 1 d. none of these

Ans. a

Q12. For the function $f(x) = x + \frac{1}{x}$

a. x = 1 is a point of maximum b. x = -1 is a point of minimum

c. maximum value > minimum value d. maximum value < minimum value

Ans. d

Q13. Function $f(x) = a^x$ is increasing on R, if

a. $a > 0$ b. $a < 0$ c. $0 > a < 1$ d. $a > 1$

Ans. d

Q14. Function $f(x) = \log_a x$ is increasing on R, if

a. $0 < a < 1$ b. $a > 1$ c. $a < 1$ d. $a > 1$

Ans. b

Q15. $\int_1^e \log x \, dx =$

a. 1 b. e – 1 c. e + 1 d. 0

Ans. a

Q16. $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ is equal to

a. $\frac{\pi}{12}$ b. $\frac{\pi}{6}$ c. $\frac{\pi}{4}$ d. $\frac{\pi}{3}$

Ans. a

Q17. $\int_0^3 \frac{3x+1}{x^2+9} dx =$

a. $\frac{\pi}{12} + \log(2\sqrt{2})$ b. $\frac{\pi}{2} + \log(2\sqrt{2})$ c. $\frac{\pi}{6} + \log(2\sqrt{2})$ d. $\frac{\pi}{3} + \log(2\sqrt{2})$

Ans. a

Q18. The value of the integral $\int_0^\infty \frac{x}{(1+x)(1+x^2)} dx$ is

a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{3}$

Ans. B

ASSERATION-REASON BASED QUESTIONS

(In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices).

A.Both **A** and **R** are true. **R** is correct explaination of **A**

B. Both **A** and **R** are true. **R** is NOT correct explaination of **A**.

C.A is true but **R** is false

D.A is false but **R** is true.

Q19. Assertion (A): Function $f(x) = \log_a x$ is increasing on R, if $a > 1$

Reasoning (R): Function is increasing if $f'(x) > 0$

Ans. a

Q20. Assertion (A): Minimum value of $f(x) = x^2 - 2x + 1$ is 0

Reasoning (R) : At $x=1$, $f''(x) > 0$

Ans. a

Section-B

(This section comprises of very short answer type- question (VSA) of 2 marks each).

Q21. Evaluate $\int \frac{x^2}{1+x^3} dx$ **(2)**

Ans. $\frac{1}{3} \log (1 + x^3)$

Q22. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$, find AB and BA. **(2)**

OR

If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find f(A) if $f(x) = x^2 - 2x + 3$.

Q23 A can run 40 metres while B run 50 metres in the same time. In a 1000 m race, find by how much distance B beats A? **(2)**

Ans. 200 m

Q24. Two pipes A and B can fill the tank in 24 minutes and 36 minutes respectively. If both the pipes are opened together, then find the time in which tank will be filled completely

Ans. 15 min 24 sec.

Q25. If $xy^2 = 1$, prove that $2\frac{dy}{dx} + y^3 = 0$

Section-C

(This section comprises of short answer type- question (SA) of 3 marks each).

Q26. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

Ans. $f'(x) > 0$.

Q27. In what ratio must rice at Rs. 45 per kg is mixed with rice at Rs. 60 per kg so

that the mixture be worth Rs. 54 per kg?

OR

In 2 hours a boat covers a certain distance in a river downstream at 17 km/h and returns back at 9 km/h. Find the speed of the stream **(3)**

Q28 If xy log (x + y) = 1, prove that $\frac{dy}{dx} = -\frac{y(x^2y+x+y)}{x(xy^2+x+y)}$.

Q29. $\int \frac{1}{x(x^3+1)} dx$.

Hint: Put $x^3 = t$

Q.30 **If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.**

Hint: $u+v= a^b$, **take log separately and solve**

Q31. Evaluate $\int \frac{dx}{x^2-16}$

Hint: $\int \frac{dx}{x^2-a^2} = \log\left(\frac{x-a}{x+a}\right)$

Section-D

(This section comprises of long answer type- question (LSA) of 5 marks each).

Q32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, **find A^{-1}. Hence using A^{-1}, solve the system of equations**

$2x + 3y + 5z = 11$, $3z + 2y - 4z = -5$, $x + y - 2z = -3$.

Q33. A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of Rs. 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for Rs. 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for Rs. 70. Using matrix method, find the cost of each variety of pen.

Hint: $x+y+z=21$, $4x+3y+2z=60$, $6x+2y+3z=70$

Q34. $\int \frac{3x+1}{(x-2)^2(x+2)} dx$?

Hint: Repeated factor

Use partial fraction: Ans. $\frac{5}{16}\log(x-2) - \frac{7}{4(x-2)} - \frac{5}{16}\log(x+2)$

OR

A runs 4 times as fast as B. If A gives B a start of 60 m, how far must the goal on the race course be so that A and B reach it at the same time?

Q35. **Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere**

Section-E

(This Section comprises of 1 case-study/passage- based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks.Internal choice is provided in 2 marks question.)

Q36. Three students Ram, Mohan and Ankit go to a shop to buy stationary. Ram purchases 2 dozen notebooks, 1 dozen pens and 4 pencils, mohan purchases 1 dozen notebook, 6 pens and 8 pencils and Ankit purchases 6 notebooks, 4 pens and 6 pencils. A notebook costs Rs. 15, a pen costs Rs. 4.50 and a Pencil costs Rs. 1.50.

Let A and B be the matrices representing the number of items purchases by the three students and the prices of the items respectively.

Based on the above information, answer the following questions:

(i) Find the order of matrix B representing the price of three items is
Ans. 1x3

(ii) Find the order of matrix A representing item purchased by three students is
Ans 3x1

(iii) calculate the bill amount of Ram

Q37. Suman is rowing a boat. She takes 6 hours to row 48 km upstream whereas she takes 3

hours to go the same distance downstream.

```Based on the above information, answer the following questions.

(i) What is the speed of rowing in still water?

Ans. 12 km/h

(ii) What is the speed of the stream?

Ans. 4 km/h

(iii) What is her average speed?

Ans10 $\frac{2}{3}$ km/h

Q38. A particle is moving along the curve represented by the polynomial $f(x) = (x - 1)(x - 2)^2$,

Based on the above information, answer the following questions:

(i) The critical points of polynomial f(x) are:

Ans. 2, $\frac{4}{3}$

(ii) The interval where f(x) is strictly increasing is

Ans. $\left(-\infty, \frac{4}{3}\right] \cup [2, \infty]$

(iii) The internal where f(x) is strictly decreasing is

Ans. $\left[\frac{4}{3}, 2\right]$
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